

CHAPTER 3

Applications of the Derivative

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CHAPTER 3

Applications of the Derivative

Section 3.1 Increasing and Decreasing Functions

Solutions to Even-Numbered Exercises

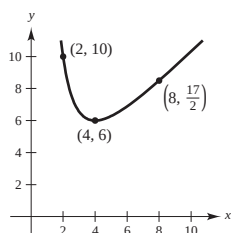
2. $f(x) = x + \frac{32}{x^2} = x + 32x^{-2}$

$$f'(x) = 1 - 64x^{-3} = \frac{x^3 - 64}{x^3}$$

At $(2, 10)$, f is decreasing since $f'(2) = -7$.

At $(4, 6)$, f has a critical number since $f'(4) = 0$.

At $(8, \frac{17}{2})$, f is increasing since $f'(8) = \frac{7}{8}$.



4. $f(x) = -3x\sqrt{x+1} = -3x(x+1)^{1/2}$

$$f'(x) = -3x \left[\frac{1}{2}(x+1)^{-1/2}(1) \right] + (x+1)^{1/2}(-3)$$

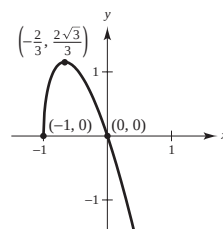
$$= -\frac{3}{2}(x+1)^{-1/2}[x + 2(x+1)]$$

$$= -\frac{3}{2}(x+1)^{-1/2}(3x+2) = -\frac{3(3x+2)}{2\sqrt{x+1}}$$

At $(-1, 0)$, f has a critical number since $f'(-1)$ is undefined.

At $(-2/3, 2\sqrt{3}/3)$, f has a critical number since $f'(-2/3) = 0$.

At $(0, 0)$, f is decreasing since $f'(0) = -3$.



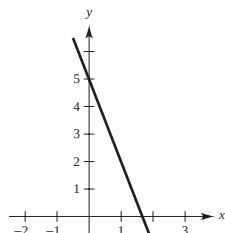
6. $f' = \frac{3}{4}x^2 - 3 = \frac{3}{4}(x^2 - 4)$

f has critical numbers at $x = \pm 2$. Moreover, f is increasing on $(-\infty, -2)$, $(2, \infty)$ and decreasing on $(-2, 2)$.

10. $f(x) = 5 - 3x$

$$f'(x) = -3$$

Since the derivative is negative for all x , the function is decreasing for all x . Thus, there are no critical numbers. f is decreasing on $(-\infty, \infty)$.



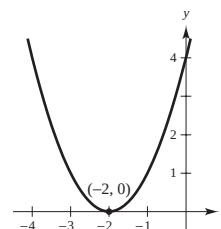
8. $f' = \frac{(x+1)(2x) - (x^2)(1)}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$

f has critical numbers at $x = 0, -2$ and has a discontinuity at $x = -1$. Moreover, f is increasing on $(-\infty, -2)$, $(0, \infty)$ and decreasing on $(-2, -1)$, $(-1, 0)$.

12. $g(x) = (x+2)^2$

$$g'(x) = 2(x+2) = 0$$

Critical number: $x = -2$



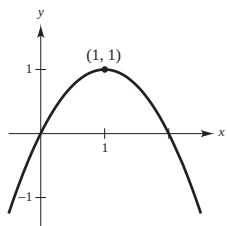
Interval	$-\infty < x < -2$	$-2 < x < \infty$
Sign of g'	$g' < 0$	$g' > 0$
Conclusion	Decreasing	Increasing

14. $y = -(x^2 - 2x) = 2x - x^2$

$$y' = 2 - 2x = 0$$

Critical number: $x = 1$

Interval	$-\infty < x < 1$	$1 < x < \infty$
Sign of y'	$y' > 0$	$y' < 0$
Conclusion	Increasing	Decreasing

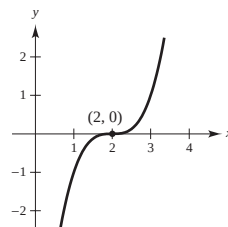


16. $y = (x - 2)^3$

$$y' = 3(x - 2)^2 = 0$$

Critical number: $x = 2$

Interval	$-\infty < x < 2$	$2 < x < \infty$
Sign of y'	$y' > 0$	$y' > 0$
Conclusion	Increasing	Increasing

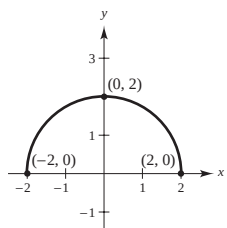
 y is increasing on $(-\infty, \infty)$.

18. $f(x) = \sqrt{4 - x^2}$

$$f'(x) = \frac{1}{2}(4 - x^2)^{-1/2}(-2x) = -\frac{x}{\sqrt{4 - x^2}}$$

Domain: $-2 \leq x \leq 2$ Critical numbers: $x = 0, x = \pm 2$

Interval	$-2 < x < 0$	$0 < x < 2$
Sign of f'	$f' > 0$	$f' < 0$
Conclusion	Increasing	Decreasing

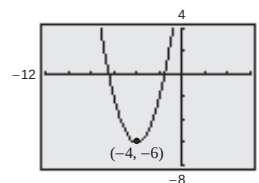


20. $f(x) = x^2 + 8x + 10$

$$f'(x) = 2x + 8 = 0$$

Critical number: $x = -4$

Interval	$-\infty < x < -4$	$-4 < x < \infty$
Sign of f'	$f' < 0$	$f' > 0$
Conclusion	Decreasing	Increasing

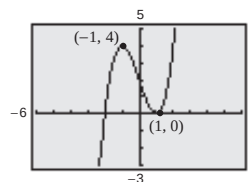


22. $f(x) = x^3 - 3x + 2 = (x - 1)^2(x + 2)$

$$f'(x) = 3x^2 - 3 = 3(x + 1)(x - 1)$$

Critical numbers: $x = \pm 1$

Interval	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of f'	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion	Increasing	Decreasing	Increasing

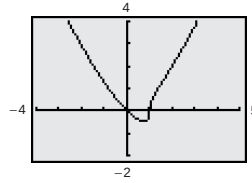


24. $h(x) = x(x - 1)^{1/3}$

$$\begin{aligned} h'(x) &= x \frac{1}{3}(x - 1)^{-2/3} + (x - 1)^{1/3} \\ &= \frac{1}{3}(x - 1)^{-2/3}(x + 3(x - 1)) \\ &= \frac{4x - 3}{3(x - 1)^{2/3}} \end{aligned}$$

Critical numbers: $x = \frac{3}{4}, 1$

Interval	$-\infty < x < \frac{3}{4}$	$\frac{3}{4} < x < 1$	$1 < x < \infty$
Sign of h'	$h' < 0$	$h' > 0$	$h' > 0$
Conclusion	Decreasing	Increasing	Increasing

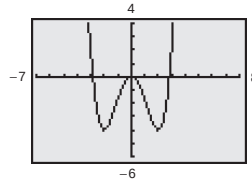


26. $f(x) = \frac{1}{4}x^4 - 2x^2$

$$f'(x) = x^3 - 4x = x(x - 2)(x + 2)$$

Critical numbers: $x = 0, 2, -2$

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of f'	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion	Decreasing	Increasing	Decreasing	Increasing

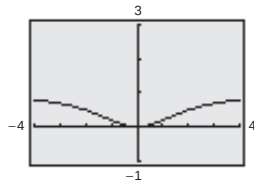


28. $f(x) = y = \frac{x^2}{x^2 + 4}$

$$y' = \frac{(x^2 + 4)(2x) - (x^2)(2x)}{(x^2 + 4)^2} = \frac{8x}{(x^2 + 4)^2}$$

Critical number: $x = 0$

Interval	$-\infty < x < 0$	$0 < x < \infty$
Sign of f'	$f' < 0$	$f' > 0$
Conclusion	Decreasing	Increasing



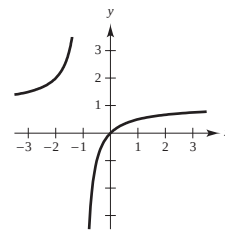
30. $f(x) = \frac{x}{x + 1}$

$$f'(x) = \frac{(x + 1)(1) - (x)(1)}{(x + 1)^2} = \frac{1}{(x + 1)^2}$$

No critical numbers

 Discontinuity: $x = -1$

Interval	$-\infty < x < -1$	$-1 < x < \infty$
Sign of f'	$f' > 0$	$f' > 0$
Conclusion	Increasing	Increasing



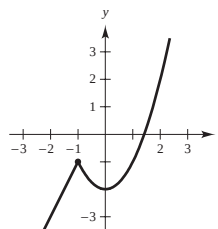
$$32. y = f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x < -1 \\ 2x, & x > -1 \end{cases}$$

$f'(-1)$ is undefined.

Critical numbers: $x = -1, 0$

Interval	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < \infty$
Sign of f'	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion	Increasing	Decreasing	Increasing



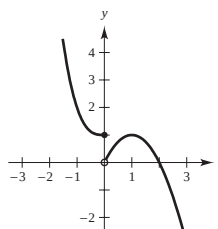
$$34. y = f(x) = \begin{cases} -x^3 + 1, & x \leq 0 \\ -x^2 + 2x, & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -3x^2, & x < 0 \\ -2x + 2, & x > 0 \end{cases}$$

$f'(0)$ is undefined.

Critical numbers: $x = 0, 1$

Interval	$-\infty < x < 0$	$0 < x < 1$	$1 < x < \infty$
Sign of f'	$f' < 0$	$f' > 0$	$f' < 0$
Conclusion	Decreasing	Increasing	Decreasing



36. As the temperature increases, the average velocity (the curve's peak) and the spread of velocities increases. For each given temperature, the number of N_2 molecules increases for the first half of the interval of velocities and decreases for the second half of the interval.

$$38. s(t) = -16t^2 + 64t, \quad 0 \leq t \leq 4$$

$$s'(t) = -32t + 64 = 0$$

The critical number is $t = 2$. Therefore, the ball is moving up on the interval $(0, 2)$ and moving down on $(2, 4)$.

$$40. P = 2.36x - \frac{x^2}{25,000} - 3500 \quad 0 \leq x \leq 50,000$$

$$(a) P' = 2.36 - \frac{1}{12,500}x = 0$$

$$x = (2.36)(12,500) = 29,500$$

- (b) You should charge the price that yields sales of $x = 29,500$ bags of popcorn.

Section 3.2 Extrema and the First-Derivative Test

$$2. f(x) = x^2 + 8x + 10$$

$$f'(x) = 2x + 8 = 2(x + 4)$$

Critical number: $x = -4$

Interval	$(-\infty, -4)$	$(-4, \infty)$
Sign of f'	-	+
f	Decreasing	Increasing

Relative minimum: $(-4, -6)$

$$4. f(x) = -4x^2 + 4x + 1$$

$$f'(x) = -8x + 4 = -4(2x - 1)$$

Critical number: $x = \frac{1}{2}$

Interval	$(-\infty, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
Sign of f'	$f' > 0$	$f' < 0$
Conclusion	Increasing	Decreasing

Relative maximum: $(\frac{1}{2}, 2)$

6. $g(x) = \frac{1}{5}x^5 - x = \frac{1}{5}(x^5 - 5x)$

$$g'(x) = \frac{1}{5}(5x^4 - 5) = x^4 - 1$$

Critical numbers: $x = \pm 1$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of g'	+	-	+
g	Increasing	Decreasing	Increasing

Relative maximum: $(-1, \frac{4}{5})$

Relative minimum: $(1, -\frac{4}{5})$

10. $f(x) = x^4 - 32x + 4$

$$f'(x) = 4x^3 - 32 = 4(x^3 - 8)$$

Critical number: $x = 2$

Interval	$(-\infty, 2)$	$(2, \infty)$
Sign of f'	-	+
f	Decreasing	Increasing

Relative minimum: $(2, -44)$

14. $f(t) = (t - 1)^{1/3}$

$$f'(t) = \frac{1}{3}(t - 1)^{-2/3} = \frac{1}{3\sqrt[3]{(t - 1)^2}}$$

Critical number: $t = 1$

Interval	$(-\infty, 1)$	$(1, \infty)$
Sign of f'	+	+
f	Increasing	Increasing

No relative extrema

8. $h(x) = 2(x - 3)^3$

$$h'(x) = 6(x - 3)^2$$

Critical number: $x = 3$

Interval	$(-\infty, 3)$	$(3, \infty)$
Sign of h'	+	+
h	Increasing	Increasing

No relative extrema

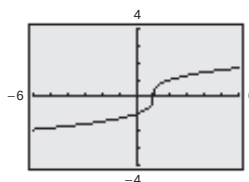
12. $f(x) = x^4 - 12x^3$

$$f'(x) = 4x^3 - 36x^2 = 4x^2(x - 9)$$

Critical numbers: $x = 0, 9$

Interval	$(-\infty, 0)$	$(0, 9)$	$(9, \infty)$
Sign of f'	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion	Decreasing	Decreasing	Increasing

Relative minimum: $(9, -2187)$



16. $f(x) = x + \frac{1}{x}$

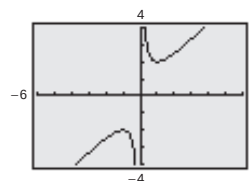
$$f'(x) = 1 - \frac{1}{x^2} = 0$$

Critical numbers: $x = \pm 1$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of f'	+	-	-	+
f	Increasing	Decreasing	Decreasing	Increasing

Relative maximum: $(-1, -2)$

Relative minimum: $(1, 2)$



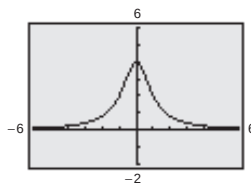
18. $f(x) = \frac{4}{x^2 + 1}$

$$f'(x) = \frac{-8x}{(x^2 + 1)^2} = 0$$

Critical number: $x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of f'	+	-
f	Increasing	Decreasing

Relative maximum: $(0, 4)$



20. $f(x) = \frac{1}{3}(2x + 5)$, $[0, 5]$

$$f'(x) = \frac{2}{3}$$

No critical numbers

x-value	Endpoint $x = 0$	Endpoint $x = 5$
$f(x)$	$\frac{5}{3}$	5
Conclusion	Minimum	Maximum

22. $f(x) = x^2 + 2x - 4$, $[-1, 1]$

$$f'(x) = 2x + 2$$

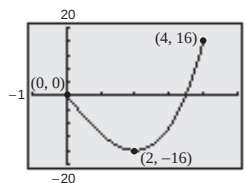
Critical number: $x = -1$ (also an endpoint)

x-value	Endpoint $x = -1$	Endpoint $x = 1$
$f(x)$	-5	-1
Conclusion	Minimum	Maximum

24. $f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 0$

Critical numbers: $x = \pm 2$

x-value	Endpoint $x = 0$	Critical $x = 2$	Endpoint $x = 4$
$f(x)$	0	-16	16
Conclusion		Minimum	Maximum



26. $h(t) = \frac{t}{t - 2}$, $[3, 5]$

$$h'(t) = \frac{-2}{(t - 2)^2}$$

No critical numbers

t-value	Endpoint $t = 3$	Endpoint $t = 5$
$h(t)$	3	$\frac{5}{3}$
Conclusion	Maximum	Minimum

28. $g(t) = \frac{t^2}{t^2 + 3}$, $[-1, 1]$

$$g'(t) = \frac{6t}{(t^2 + 3)^2}$$

Critical number: $t = 0$

t-value	Endpoint $t = -1$	Critical $t = 0$	Endpoint $t = 1$
$g(t)$	$\frac{1}{4}$	0	$\frac{1}{4}$
Conclusion	Maximum	Minimum	Maximum

30. $g(x) = 4\left(1 + \frac{1}{x} + \frac{1}{x^2}\right) = 4(1 + x^{-1} + x^{-2})$, $[-4, 5]$

$$g'(x) = 4(-x^{-2} - 2x^{-3}) = 4\left(-\frac{1}{x^2} - \frac{2}{x^3}\right) = -4\left(\frac{x + 2}{x^3}\right)$$

Critical number: $x = -2$

Discontinuity: $x = 0$

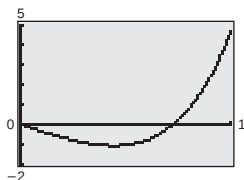
No maximum

x-value	Endpoint $x = -4$	Critical $x = -2$	Discontinuity $x = 0$	Endpoint $x = 5$
$g(x)$	3.25	3	Undefined	4.96
Conclusion		Minimum		

32. $f(x) = 3.2x^5 + 5x^3 - 3.5x, \quad [0, 1]$

Maximum: (1, 4.7)

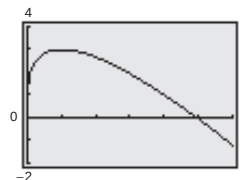
Minimum: (0.4398, -1.0613)



34. $f(x) = 4\sqrt{x} - 2x + 1, \quad [0, 6]$

Maximum: (1, 3)

Minimum: (6, -1.202)



36. $f(x) = \frac{8}{x+1}, \quad [0, \infty)$

$$f'(x) = \frac{-8}{(x+1)^2}$$

 No critical numbers; f is decreasing on $[0, \infty)$

 Maximum: $f(0) = 8$

38. $f(x) = 8 - \frac{4x}{x^2 + 1}, \quad [0, \infty)$

$$f'(x) = -\frac{(x^2 + 1)(4) - 4x(2x)}{(x^2 + 1)^2} = \frac{4x^2 - 4}{(x^2 + 1)^2} = 0$$

 Critical number: $x = 1$

 Maximum: $f(0) = 8$

 Minimum: $f(1) = 6$

40. $f(x) = \frac{1}{x^2 + 1}, \quad [0, 3]$

$$f'(x) = \frac{-2x}{(x^2 + 1)^2}$$

$$f''(x) = \frac{2(3x^2 - 1)}{(x^2 + 1)^3}$$

$$f'''(x) = \frac{24x(1 - x^2)}{(x^2 + 1)^4}$$

 Critical numbers for f'' in $[0, 3]$: $x = 0, x = 1$

x-value	Endpoint $x = 0$	Critical $x = 1$	Endpoint $x = 3$
$ f''(x) $	2	$\frac{1}{2}$	$\frac{13}{250}$
Conclusion	Maximum		

42. $f(x) = \frac{1}{x^2}, \quad [1, 2]$

$$f'(x) = -\frac{2}{x^3}$$

$$f''(x) = \frac{6}{x^4}$$

$$f'''(x) = -\frac{24}{x^5}$$

$$f^{(4)}(x) = \frac{120}{x^6}$$

$$f^{(5)}(x) = -\frac{720}{x^7}$$

 No critical numbers of $f^{(4)}$

x-value	Endpoint $x = 1$	Endpoint $x = 2$
$ f^{(4)}(x) $	120	$\frac{15}{8}$
Conclusion	Maximum	

44. $x = \frac{k}{p^3}, \quad p > 1$

$$8 = \frac{k}{(10)^3} \Rightarrow k = 8000$$

$$x = \frac{8000}{p^3} \Rightarrow p = \frac{20}{\sqrt[3]{x}}$$

$$C = 4x + 100$$

$$P = xp - C = x\left(\frac{20}{\sqrt[3]{x}}\right) - (4x + 100) = 20x^{2/3} - 4x - 100$$

$$\frac{dP}{dx} = \frac{40}{3}x^{-1/3} - 4 = 0 \text{ when } x = \frac{1000}{27} \approx 37 \text{ units.}$$

Since P is increasing on $(1, 1000/27)$ and decreasing on $(1000/27, \infty)$, the profit is maximum when $x = 1000/27 \approx 37$ units and the price is \$6.00.

46. $v = k(R - r)r^2, \quad 0 \leq r < R$

$$= k(Rr^2 - r^3)$$

$$\frac{dv}{dr} = k(2Rr - 3r^2) = kr(2R - 3r)$$

Critical numbers: $r = 0, r = \frac{2R}{3}$

r -value	Endpoint $r = 0$	Critical $r = (2R)/3$	Endpoint $r \rightarrow R$
v	0	$(4kR^3)/27$	$v \rightarrow 0$
Conclusion		Maximum	

The maximum air velocity occurs when $r = 2R/3$.

48. (a) 1970: 2500 per 1000 women

(b) 1985-1990 most rapidly
1975-1980 most slowly

(c) 1970-1975 most rapidly
1980-1985 most slowly

(d) Answers will vary.

Section 3.3 Concavity and the Second-Derivative Test

2. $f(x) = -x^3 + 3x^2 - 2$

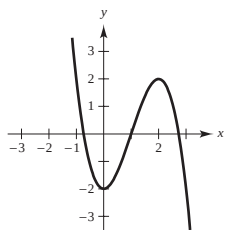
$$f'(x) = -3x^2 + 6x$$

$$f''(x) = -6x + 6$$

$$f''(x) = 0 \text{ when } x = 1.$$

Concave upward on $(-\infty, 1)$

Concave downward on $(1, \infty)$



4. $f(x) = \frac{x^2 + 4}{4 - x^2}$

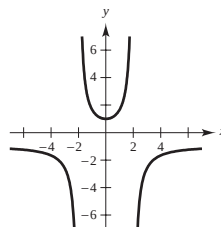
$$f'(x) = \frac{(4 - x^2)2x - (x^2 + 4)(-2x)}{(4 - x^2)^2} = \frac{16x}{(4 - x^2)^2}$$

$$f''(x) = \frac{(4 - x^2)^2 16 - 16x \cdot 2(4 - x^2)(-2x)}{(4 - x^2)^4} = \frac{-16(4 + 3x^2)}{(4 - x^2)^3}$$

f'' is undefined at $x = \pm 2$.

Concave upward on $(-2, 2)$

Concave downward on $(-\infty, -2)$ and $(2, \infty)$



6. $f(x) = \frac{x^2}{x^2 + 1}$

$$f'(x) = \frac{2x}{(x^2 + 1)^2}$$

$$f''(x) = \frac{-2(3x^2 - 1)}{(x^2 + 1)^3}$$

$$f''(x) = 0 \text{ when } x = \pm \frac{1}{\sqrt{3}} \approx \pm 0.5774$$

Concave upward on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Concave downward on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$

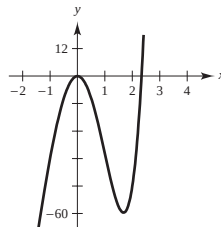
8. $f(x) = x^5 + 5x^4 - 40x^2$

$$f'(x) = 5x^4 + 20x^3 - 80x$$

$$f''(x) = 20x^3 + 60x^2 - 80$$

Concave upward on $(1, \infty)$

Concave downward on $(-\infty, 1)$



10. $f(x) = (x - 5)^2$

$$f'(x) = 2(x - 5) = 0$$

Critical number: $x = 5$

$$f''(x) = 2$$

$$f''(5) = 2 > 0$$

Thus, $(5, 0)$ is a relative minimum.

12. $f(x) = x^4 - 4x^3 + 2$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) = 0$$

Critical numbers: $x = 0, 3$

$$f''(x) = 12x^2 - 24x$$

$$f''(0) = 0 \quad \text{Test fails}$$

$$f''(3) > 0$$

 $(3, -24)$ is a relative minimum. $(0, 3)$ is not a relative extremum.

14. $f(x) = x + \frac{4}{x}$

$$f'(x) = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2}$$

Critical numbers: $x = \pm 2$

$$f''(x) = \frac{8}{x^3}$$

$$f''(2) = 1 > 0$$

$$f''(-2) = -1 < 0$$

Thus, $(2, 4)$ is a relative minimum and $(-2, -4)$ is a relative maximum.

16. $f(x) = \sqrt{4 - x^2}$ Domain: $-2 \leq x \leq 2$

$$f'(x) = \frac{-x}{\sqrt{4 - x^2}}$$

$$f''(x) = \frac{-4}{(4 - x^2)^{3/2}}$$

Critical numbers: $x = 0$

$$f''(0) < 0$$

 $(0, 2)$ is a relative maximum. $[(2, 0)$ and $(-2, 0)$ are absolute minimums.]

18. $f(x) = \frac{x}{x^2 - 1}$

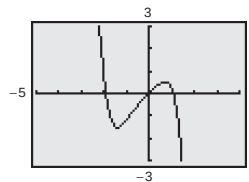
$$f'(x) = -\frac{1 + x^2}{(x^2 - 1)^2}$$

$$f''(x) = \frac{2x(x^2 + 3)}{(x^2 - 1)^3}$$

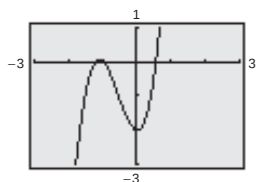
No critical numbers

No relative extrema

20. $f(x) = -\frac{1}{3}x^5 - \frac{1}{2}x^4 + x$

Relative maximum: $(0.6830, 0.5247)$ Relative minimum: $(-1.4128, -1.5286)$

22.

 $(-1.11, 0.057)$ is a relative maximum. $(0, -2)$ is a relative minimum.

26. $f' < 0$, (decreasing)

$$f'' > 0$$
, (concave upward)

24. $f' > 0$, (increasing)

$$f'' < 0$$
, (concave downward)

28. $f(x) = x(6 - x)^2 = x^3 - 12x^2 + 36x$

$$f'(x) = 3x^2 - 24x + 36$$

$$f''(x) = 6x - 24 = 0 \text{ when } x = 4.$$

$$f''(x) < 0 \text{ on } (-\infty, 4)$$

$$f''(x) > 0 \text{ on } (4, \infty)$$

Thus, $(4, 16)$ is an inflection point.

30. $f(x) = x^4 - 18x^2 + 5$

$$f'(x) = 4x^3 - 36x$$

$$f''(x) = 12x^2 - 36 = 12(x^2 - 3) = 0 \text{ when } x = \pm\sqrt{3}.$$

$$f''(x) > 0 \text{ on } (-\infty, -\sqrt{3})$$

$$f''(x) < 0 \text{ on } (-\sqrt{3}, \sqrt{3})$$

$$f''(x) > 0 \text{ on } (\sqrt{3}, \infty)$$

Thus, $(-\sqrt{3}, -40)$ and $(\sqrt{3}, -40)$ are inflection points.

34. $f(t) = (1-t)(t-4)(t^2-4)$

$$f'(t) = -4t^3 + 15t^2 - 20$$

$$f''(t) = -6t(2t-5)$$

Inflection points: $(0, 16), (\frac{5}{2}, \frac{81}{16})$

38. $f(x) = x^3 - \frac{3}{2}x^2 - 6x$

$$f'(x) = 3x^2 - 3x - 6 = 3(x+1)(x-2)$$

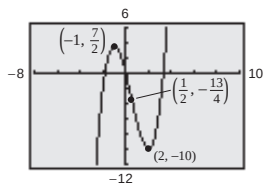
Critical numbers: $x = -1, x = 2$

$$f''(x) = 6x - 3$$

$$f''(-1) = -9 < 0 \Rightarrow (-1, \frac{7}{2}) \text{ relative maximum}$$

$$f''(2) = 9 > 0 \Rightarrow (2, -10) \text{ relative minimum}$$

Point of inflection is $(\frac{1}{2}, -\frac{13}{4})$.



32. $f(x) = -4x^3 - 8x^2 + 32$

$$f'(x) = -12x^2 - 16x$$

$$f''(x) = -24x - 16 = 0 \Rightarrow x = -\frac{2}{3}$$

Since f'' changes sign at $x = -\frac{2}{3}$, $(-\frac{2}{3}, \frac{800}{27})$ is a point of inflection.

36. $f(x) = x^3 - 3x$

$$f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$$

Critical numbers: $x = -1, 1$

$$f''(x) = 6x$$

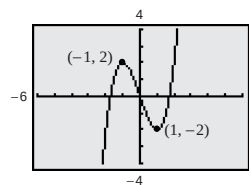
$$f''(-1) = -6 < 0$$

Thus $(-1, 2)$ is a relative maximum.

$$f''(1) = 6 > 0$$

Thus $(1, -2)$ is a relative minimum.

Point of inflection: $(0, 0)$



40. $f(x) = 2x^4 - 8x + 3$

$$f'(x) = 8x^3 - 8 = 8(x^3 - 1)$$

Critical number: $x = 1$

$$f''(x) = 24x^2$$

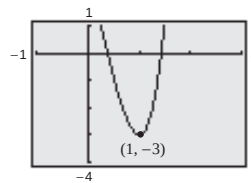
$$f''(1) = 24 > 0$$

Thus, $(1, -3)$ is a relative minimum.

$$f''(x) = 24x^2 = 0 \text{ when } x = 0.$$

$$f''(x) > 0 \text{ on } (-\infty, 0) \text{ and on } (0, \infty)$$

Thus, there are no inflection points.



42. $g(x) = (x - 6)(x + 2)^3$

$$g'(x) = (x - 6)[3(x + 2)^2] + (x + 2)^3(1)$$

$$= (x + 2)^2[3(x - 6) + (x + 2)] = (x + 2)^2(4x - 16)$$

 Critical numbers: $x = -2, 4$

$$g''(x) = (x + 2)^2(4) + (4x - 16)[2(x + 2)]$$

$$= 2(x + 2)[2(x + 2) + (4x - 16)]$$

$$= 2(x + 2)(6x - 12) = 12(x + 2)(x - 2)$$

$$g''(-2) = 0 \quad (\text{Test fails})$$

$$g''(4) = 144 > 0$$

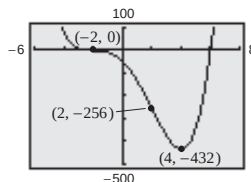
 Thus, $(4, -432)$ is a relative minimum. By the First-Derivative Test, $(-2, 0)$ is not a relative extremum.

$$g''(x) = 12(x + 2)(x - 2) = 0 \text{ when } x = \pm 2.$$

$$g''(x) > 0 \text{ on } (-\infty, -2)$$

$$g''(x) < 0 \text{ on } (-2, 2)$$

$$g''(x) > 0 \text{ on } (2, \infty)$$

 Thus, $(-2, 0)$ and $(2, -256)$ are points of inflection.


44. $g(x) = x\sqrt{9 - x}$

$$g'(x) = \frac{18 - 3x}{2\sqrt{9 - x}}$$

$$g''(x) = \frac{3(x - 12)}{4(9 - x)^{3/2}}$$

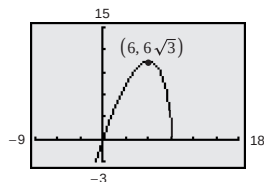
 Critical number: $x = 6$

$$g''(6) < 0$$

 Relative maximum: $(6, 6\sqrt{3}) \approx (6, 10.3923)$
 $g'' < 0$ for all x in domain.

 Concave downward on $(-\infty, 9)$.

No points of inflection



46. $f(x) = \frac{2}{x^2 - 1}$

$$f'(x) = \frac{-4x}{(x^2 - 1)^2}$$

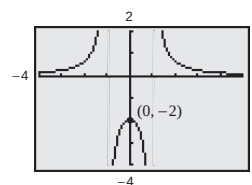
$$f''(x) = \frac{4(3x^2 + 1)}{(x^2 - 1)^3}$$

 Critical number: $x = 0$

$$f''(0) < 0$$

 $(0, -2)$ is a relative maximum.

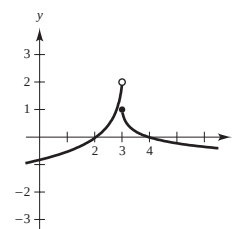
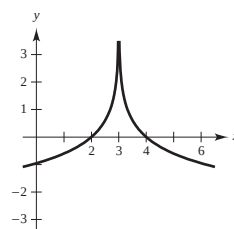
No points of inflection


 48.

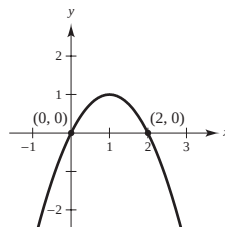
Function	First Derivative	Second Derivative
$f(2) = 0$	$f'(x) > 0, x < 3$	$f''(x) > 0, x \neq 3$
$f(4) = 0$	$f'(3)$ is undefined.	
	$f'(x) < 0, x > 3$	

$f(2) = 0$ $f'(x) > 0, x < 3$ $f''(x) > 0, x \neq 3$
 $f(4) = 0$ $f'(3)$ is undefined.
 $f'(x) < 0, x > 3$

The function has x -intercepts at $(2, 0)$ and $(4, 0)$. On $(-\infty, 3)$, f is increasing and on $(3, \infty)$, f is decreasing. f has either a relative maximum at $x = 3$ or a discontinuity at $x = 3$. Also, f is concave upward on $(-\infty, 3)$ and $(3, \infty)$.



50. (a) f' is positive on $(0, 2)$ where f is increasing.
 (b) f' is negative on $(-\infty, 0)$ and $(2, \infty)$ where f is decreasing.
 (c) f' is increasing on $(-\infty, 1)$ where f is concave upward.
 (d) f' is decreasing on $(1, \infty)$ where f is concave downward.



52. $R' = -\frac{4}{9}(3x^2 - 18x)$

$$R'' = -\frac{4}{9}(6x - 18) = 0 \text{ when } x = 3.$$

$$R'' > 0 \text{ on } (0, 3)$$

$$R'' < 0 \text{ on } (3, 5)$$

Since $(3, 36)$ is a point of inflection, it is the point of diminishing returns.

54. $C = 0.002x^3 + 20x + 500$

$$\bar{C} = 0.002x^2 + 20 + \frac{500}{x}$$

$$\bar{C}' = 0.004x - \frac{500}{x^2} = 0$$

$$x^3 = \frac{500}{0.004}$$

$$x = 50 \text{ units}$$

56. $N(t) = \frac{20t^2}{4 + t^2}, \quad 0 \leq t \leq 4$

We need to determine when $N'(t)$ is greatest.

$$N'(t) = \frac{(4 + t^2)(40t) - (20t^2)(2t)}{(4 + t^2)^2}$$

$$= 160t(4 + t^2)^{-2}$$

$$N''(t) = 160\{t[-2(4 + t^2)^{-3}(2t)] + (4 + t^2)^{-2}(1)\}$$

$$= 160(4 + t^2)^{-3}[-4t^2 + (4 + t^2)]$$

$$= \frac{160(4 - 3t^2)}{(4 + t^2)^3}$$

$$= 0 \text{ when } t = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3}.$$

The student is assembling components at the greatest rate when $t = (2\sqrt{3})/3$ or at approximately 8:09 P.M.

58. $S(t) = \frac{500,000t^2}{36 + t^2}$

$$S'(t) = 500,000 \left[\frac{(36 + t^2)(2t) - (t^2)(2t)}{(36 + t^2)^2} \right]$$

$$= 36,000,000t(36 + t^2)^{-2}$$

$$S''(t) = 36,000,000t[-2(36 + t^2)^{-3}(2t) + (36 + t^2)^{-2}(1)]$$

$$= 36,000,000(36 + t^2)^{-3}[-4t^2 + (36 + t^2)]$$

$$= \frac{36,000,000(36 - 3t^2)}{(36 + t^2)^3}$$

$$= 0 \text{ when } t = \pm \sqrt{12} = \pm 2\sqrt{3}.$$

The sales of this new product is increasing at the greatest rate when $t = 2\sqrt{3} \approx 3.464$ years.

60. $f(x) = -\frac{1}{20}x^5 - \frac{1}{12}x^2 - \frac{1}{3}x + 1, \quad [-2, 2]$

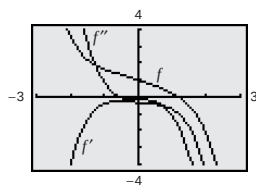
$$f'(x) = -\frac{1}{4}x^4 - \frac{1}{6}x - \frac{1}{3}$$

$$f''(x) = -x^3 - \frac{1}{6}$$

$$\text{Minimum: } (2, -1.6)$$

$$\text{Maximum: } (-2, 2.93)$$

$$\text{Point of inflection: } (-0.55, 1.16)$$



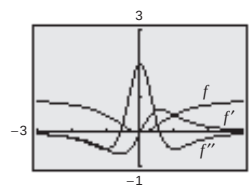
62. $f(x) = \frac{x^2}{x^2 + 1}, \quad [-3, 3]$

$$f'(x) = \frac{(x^2 + 1)(2x) - x^2(2x)}{(x^2 + 1)^2} = \frac{2x}{(x^2 + 1)^2}$$

$$f''(x) = \frac{2(1 - 3x^2)}{(x^2 + 1)^3}$$

$$\text{Relative minimum: } (0, 0)$$

$$\text{Inflection points: } \left(\frac{1}{\sqrt{3}}, \frac{1}{4}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{4}\right)$$



64. (a) $S'' > 0$
 (b) $S' > 0$ and $S'' > 0$
 (c) $S'' = 0$
 (d) $S' = 0$
 (e) $S' < 0$ and $S'' > 0$
 (f) $S' = 0$ and $S'' > 0$

66. Answers will vary.

Section 3.4 Optimization Problems

2. Let x be the first number and y be the second number. Then $x + y = S$ and $y = S - x$. Thus, the product of x and y is given by the following.

$$P = xy = x(S - x) = Sx - x^2$$

$$P' = S - 2x$$

$P' = 0$ when $x = S/2$. Since $P''(S/2) = -2 < 0$, the product is maximum when $x = y = S/2$.

6. Let x be the first number and y be the second number. Then $xy = 192$ and $y = 192/x$. The sum is given by the following.

$$S = x + 3y = x + 3\left(\frac{192}{x}\right) = x + 576x^{-1}$$

$$S' = 1 - 576x^{-2} = \frac{x^2 - 576}{x^2}$$

Critical number: $x = 24$

$$S'' = \frac{1152}{x^3}$$

Since $S''(24) > 0$, the sum is minimum when $x = 24$ and $y = 192/24 = 8$.

10. Let x be the length and y be the width of the rectangle. Then $2x + 2y = P$ and $y = \frac{1}{2}(P - 2x)$. The area is given by the following.

$$A = xy = x \cdot \frac{1}{2}(P - 2x) = \frac{1}{2}Px - x^2$$

$$A' = \frac{1}{2}P - 2x$$

$A' = 0$ when $x = P/4$. Since $A'' = -2 < 0$, A is a maximum when $x = P/4$ and $y = P/4$. This is a square!

4. Let x be the first number and y be the second number. Then $x + 2y = 100$ and $x = 100 - 2y$. Thus, the product of x and y is given by the following.

$$P = xy = (100 - 2y)y = 100y - 2y^2$$

$$P' = 100 - 4y$$

$P' = 0$ when $y = 25$. Since $P''(25) = -4 < 0$, the product is maximum when $x = 100 - 2(25) = 50$ and $y = 25$.

8. Let x be the first number and y be the second number. Then $x - y = 50$ and $x = 50 + y$. The product is given by the following.

$$P = xy = (50 + y)y = 50y + y^2$$

$$P' = 50 + 2y$$

$P' = 0$ when $y = -25$. Since $P''(-25) = 2 > 0$, the product is minimum when $x = 50 + (-25) = 25$ and $y = -25$.

12. Let x and y be the length and width of the rectangle. The area is $xy = A$ and $y = A/x$. The perimeter is given by the following.

$$P = 2x + 2y = 2x + 2\left(\frac{A}{x}\right) = 2x + 2Ax^{-1}$$

$$P' = 2 - 2Ax^{-2} = 2 - \frac{2A}{x^2} = \frac{2x^2 - 2A}{x^2}$$

$$P' = 0 \text{ when } x^2 = A, \text{ or } x = \sqrt{A}.$$

$$P''(x) = 4Ax^{-3} > 0 \Rightarrow \text{this is a minimum.}$$

Hence, $x = \sqrt{A}$, $y = \sqrt{A}$ (a square!)

14. Let x and y be the lengths shown in the figure. Then $xy = 180,000$ and $y = 180,000/x$. The perimeter is given by the following.

$$P = 2x + y = 2x + \frac{180,000}{x}$$

$$P' = 2 - \frac{180,000}{x^2} = \frac{2(x^2 - 90,000)}{x^2}$$

$$P' = 0 \text{ when } x = 300. \text{ Since } P'' = \frac{360,000}{x^3} \text{ and } P''(300) > 0,$$

$$P \text{ is minimum when } x = 300 \text{ meters and } y = \frac{180,000}{300} = 600 \text{ meters.}$$

16. Let x be the sides of the base, and y the height.

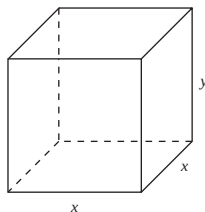
$$\text{Surface area} = 337.5 = 2x^2 + 4xy \Rightarrow y = \frac{1}{4x}[337.5 - 2x^2]$$

$$V = x^2y = x^2 \left[\frac{1}{4x}(337.5 - 2x^2) \right] = \frac{675}{8}x - \frac{x^3}{2}$$

$$V'(x) = \frac{675}{8} - \frac{3x^2}{2} = 0 \Rightarrow x = 7.5 \Rightarrow y = 7.5$$

$$V''(x) = -3x < 0 \Rightarrow x = 7.5 \text{ is a maximum.}$$

$$\text{Dimensions: } 7.5 \times 7.5 \times 7.5$$

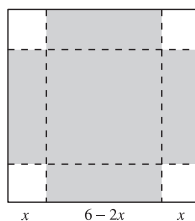


18. Let x be the length shown in the figure. Then the volume of the box is given by the following.

$$V = x(6 - 2x)^2, \quad 0 < x < 3$$

$$V' = 12(x - 1)(x - 3)$$

$V' = 0$ when $x = 3$ and $x = 1$. Since $V = 0$ when $x = 3$ and $V = 16$ when $x = 1$, we conclude that the volume is maximum when $x = 1$. The corresponding volume is $V = 16$ cubic inches.



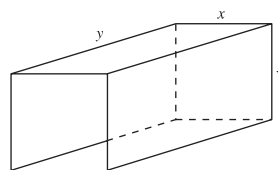
20. Let x and y be the lengths shown in the figure. Then $x^2y = 250/3$ and $y = 250/3x^2$. The surface area of the enclosure is given by the following.

$$A = 3xy + x^2 = 3x \left(\frac{250}{3x^2} \right) + x^2 = \frac{250}{x} + x^2$$

$$A' = -\frac{250}{x^2} + 2x = \frac{2x^3 - 250}{x^2}$$

$A' = 0$ when $x = 5$. The surface area is minimum when $x = 5$ meters and

$$y = \frac{250}{3(5)^2} = \frac{10}{3} \text{ meters.}$$



22. $A = (x + 3)(y + 3)$

$$36 = xy \Rightarrow y = \frac{36}{x}$$

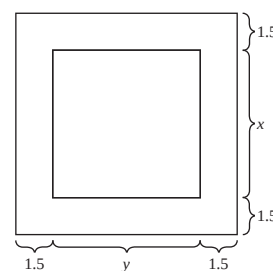
$$A = (x + 3)\left(\frac{36}{x} + 3\right)$$

$$= 36 + \frac{108}{x} + 3x + 9$$

$$\frac{dA}{dx} = -\frac{108}{x^2} + 3 = 0$$

$$x^2 = 36$$

$$x = 6, y = 6$$



By the First-Derivative Test, A is a minimum when $x = y = 6$. Thus, length = width = 9 inches.

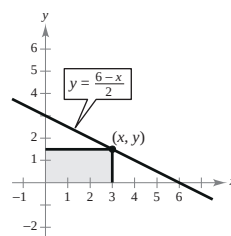
24. The area of the rectangle is

$$A = xy = x\left(\frac{6-x}{2}\right) = \frac{1}{2}(6x - x^2)$$

$$A' = \frac{1}{2}(6 - 2x).$$

$A' = 0$ when $x = 3$. Thus, A is maximum when $x = 3$ and

$$y = \frac{6-3}{2} = \frac{3}{2}.$$



26. The area is given by the following.

$$A = 2xy = 2x\sqrt{25 - x^2}$$

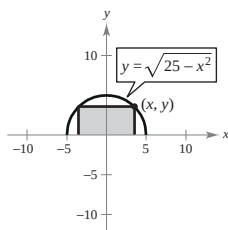
$$A' = 2\left(\frac{25 - 2x^2}{\sqrt{25 - x^2}}\right)$$

$A' = 0$ when $x = 5/\sqrt{2}$. Thus, A is maximum when the length is

$$2x = \frac{10}{\sqrt{2}} \approx 7.07$$

and the width is

$$y = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2} = \frac{5}{\sqrt{2}} \approx 3.54.$$



28. The volume of the cylinder is

$$V = \pi r^2 h = 12(1.80469) \approx 21.66$$

which implies that $h = 21.66/\pi r^2$. The surface area of the cylinder is

$$S = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \left(\frac{21.66}{\pi r^2}\right)$$

$$= 2\left(\pi r^2 + \frac{21.66}{r}\right)$$

$$S' = 2\left(2\pi r - \frac{21.66}{r^2}\right).$$

$S' = 0$ when $2\pi r^3 - 21.66 = 0$, which implies that

$$r = \sqrt[3]{\frac{21.66}{2\pi}} \approx 1.51 \text{ inches}$$

$$h = \frac{21.66}{\pi(1.51)^2} \approx 3.02 \text{ inches.}$$

(Note that in the solution, $h = 2r$.)

30. $V = \frac{1}{3}\pi x^2 h = \frac{1}{3}\pi x^2(r + \sqrt{r^2 - x^2})$ (see figure)

$$\frac{dV}{dx} = \frac{1}{3}\pi \left[\frac{-x^3}{\sqrt{r^2 - x^2}} + 2x(r + \sqrt{r^2 - x^2}) - \frac{\pi x}{3\sqrt{r^2 - x^2}}(2r^2 + 2r\sqrt{r^2 - x^2} - 3x^2) \right] = 0$$

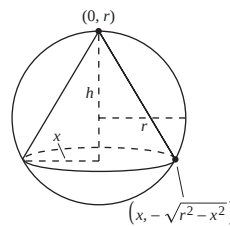
$$2r^2 + 2r\sqrt{r^2 - x^2} - 3x^2 = 0$$

$$2r\sqrt{r^2 - x^2} = 3x^2 - 2r^2$$

$$4r^2(r^2 - x^2) = 9x^4 - 12x^2r^2 + 4r^4$$

$$0 = 9x^4 - 8x^2r^2 = x^2(9x^2 - 8r^2)$$

$$x = 0, \frac{2\sqrt{2}r}{3}$$



By the First Derivative Test, the volume is a maximum when

$$x = \frac{2\sqrt{2}r}{3} \text{ and } h = r + \sqrt{r^2 - x^2} = \frac{4r}{3}.$$

Thus, the maximum volume is

$$V = \frac{1}{3}\pi \left(\frac{8r^2}{9} \right) \left(\frac{4r}{3} \right) = \frac{32\pi r^3}{81} \text{ cubic units.}$$

32. The distance between a point (x, y) on the graph and the point $(2, 1/2)$ is

$$d = \sqrt{(x - 2)^2 + \left(y - \frac{1}{2}\right)^2} = \sqrt{(x - 2)^2 + \left(x^2 - \frac{1}{2}\right)^2}$$

and we can minimize d by minimizing its square $L = d^2$.

$$L = (x - 2)^2 + \left(x^2 - \frac{1}{2}\right)^2 = x^4 - 4x + \frac{17}{4}$$

$$L' = 4x^3 - 4$$

$L' = 0$ when $x = 1$ and $y = (1)^2 = 1$. Thus, the point nearest $(2, 1/2)$ is $(1, 1)$.

34. The volume is $\frac{4}{3}\pi r^3 + \pi r^2 h = 12$, thus $h = \frac{12}{\pi r^2} - \frac{4}{3}r$.

The surface area is given by the following.

$$A = 4\pi r^2 + 2\pi r h$$

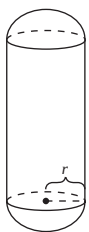
$$= 4\pi r^2 + 2\pi r \left(\frac{12}{\pi r^2} - \frac{4}{3}r \right)$$

$$= 4\pi r^2 + \frac{24}{r} - \frac{8}{3}\pi r^2$$

$$= \frac{4}{3}\pi r^2 + \frac{24}{r}$$

$$A' = \frac{8}{3}\pi r - \frac{24}{r^2} = \frac{8(\pi r^3 - 9)}{3r^2}$$

$A' = 0$ when $r = \sqrt[3]{9/\pi} \approx 1.42$ in.



36. The perimeter is given by $3x + 4y = 10$. Thus, $y = \frac{1}{4}(10 - 3x)$. The total area is given by the following.

$$A = \frac{1}{2}bh + y^2 = \frac{1}{2}x\left(\frac{\sqrt{3}x}{2}\right) + \left[\frac{1}{4}(10 - 3x)\right]^2$$

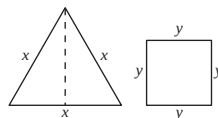
$$= \frac{\sqrt{3}}{4}x^2 + \frac{1}{16}(100 - 60x + 9x^2)$$

$$A' = \frac{\sqrt{3}}{2}x + \frac{1}{16}(-60 + 18x)$$

$$= x\left(\frac{\sqrt{3}}{2} + \frac{9}{8}\right) - \frac{15}{4}$$

$A' = 0$ when $x = \frac{30}{4\sqrt{3} + 9}$ and

$$y = \frac{1}{4}\left[10 - 3\left(\frac{30}{4\sqrt{3} + 9}\right)\right] = \frac{10\sqrt{3}}{4\sqrt{3} + 9}.$$



38. Let x and y be the length and width of the rectangle. The radius of the semicircle is $r = y/2$, and the perimeter is

$$200 = 2x + 2\pi r = 2x + 2\pi\left(\frac{y}{2}\right) = 2x + \pi y$$

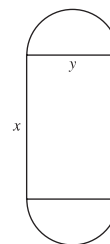
which implies that $y = (200 - 2x)/\pi$. The area of the rectangle is given by the following.

$$A = xy = x\left[\frac{200 - 2x}{\pi}\right] = \frac{2}{\pi}(100x - x^2)$$

$$A' = \frac{2}{\pi}(100 - 2x)$$

$A' = 0$ when $x = 50$. Thus, A is maximum when $x = 50$ meters and

$$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi} \text{ meters.}$$



40. Since $h^2 + w^2 = 24^2$, we have $h^2 = 24^2 - w^2$.

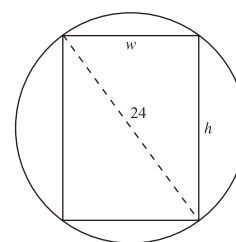
$$S = kh^2w = k(24^2 - w^2)w = k(576w - w^3)$$

$$S' = k(576 - 3w^2)$$

$S' = 0$ when $w = \sqrt{192} = 8\sqrt{3}$. Thus, S is maximum when

$$h^2 = 24^2 - 192 = 384 \Rightarrow h = \sqrt{384} = 8\sqrt{6}.$$

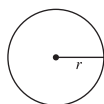
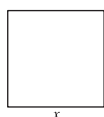
The dimensions are $w = 8\sqrt{3} \approx 13.856$ inches and $h = 8\sqrt{6} \approx 19.596$ inches.



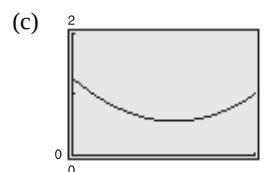
42. (a) Area = $x^2 + \pi r^2$

$$4 = 4x + 2\pi r \Rightarrow r = \frac{2 - 2x}{\pi}$$

$$A(x) = x^2 + \pi\left(\frac{2 - 2x}{\pi}\right)^2 = x^2 + \frac{(2 - 2x)^2}{\pi}$$



- (b) Domain: $0 \leq x \leq 1$



- (d) A is minimum if $x = 0.5601$ and $r = 0.28$;

- (e) A is maximum if $x = 0$ and $r = 2/\pi$ (all the wire for the circle).

Section 3.5 Business and Economics Applications

2. $R = 48x^2 - 0.02x^3$

$$R' = 96x - 0.06x^2 = x(96 - 0.06x)$$

$$R' = 0 \text{ when } x = 0 \text{ or } x = 1600.$$

Thus, R is maximum when $x = 1600$ units.

4. $R = 30x^{2/3} - 2x$

$$R' = 20x^{-1/3} - 2 = \frac{20}{\sqrt[3]{x}} - 2$$

$$R' = 0 \text{ when } x = 1000.$$

Thus, R is maximum when $x = 1000$ units.

$$6. \quad \bar{C} = 0.001x^2 + 5 + \frac{250}{x}$$

$$\bar{C}' = 0.002x - \frac{250}{x^2} = \frac{0.002x^3 - 250}{x^2}$$

$$\bar{C}' = 0 \text{ when } x = 50.$$

Thus, $\bar{C}$ is minimum when $x = 50$ units.

$$8. \quad \bar{C} = 0.02x^2 + 55x + \frac{1250}{x}$$

$$\bar{C}' = 0.04x + 55 - \frac{1250}{x^2} = 0$$

$$\frac{0.04x^3 + 55x^2 - 1250}{x^2} = 0 \Rightarrow x \approx 5 \text{ units}$$

$$10. \quad P = xp - C = x\left(\frac{60}{\sqrt{x}}\right) - (0.5x + 600) \\ = 60x^{1/2} - 0.5x - 600$$

$$P' = 30x^{-1/2} - 0.5$$

$$P' = 0 \text{ when } \frac{30}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 60 \Rightarrow x = 3600$$

By the First Derivative Test, this is a maximum.

$$x = 3600, p = \frac{60}{\sqrt{3600}} = 1 \text{ dollar}$$

$$12. \quad P = xp - C = x\left(50 - \frac{\sqrt{x}}{10}\right) - (35x + 500) \\ = 15x - \frac{1}{10}x^{3/2} - 500$$

$$P' = 15 - \frac{3}{20}x^{1/2}$$

$$P' = 0 \text{ when } 15 = \frac{3}{20}\sqrt{x}$$

$$100 = \sqrt{x}$$

$$10,000 = x.$$

By the First-Derivative Test, P is a maximum when

$$x = 10,000 \text{ units and } p = 50 - \frac{\sqrt{10,000}}{10} = \$40.$$

$$14. \quad C = x^3 - 6x^2 + 13x$$

$$\bar{C} = x^2 - 6x + 13$$

$$\bar{C}' = 2x - 6$$

$$\bar{C}' = 0 \text{ when } x = 3.$$

Thus, the average cost is minimum when $x = 3$ units and $\bar{C}(3) = \$4$ per unit.

$$C' = \text{marginal cost} = 3x^2 - 12x + 13$$

$$C'(3) = 4 = \bar{C}(3)$$

$$16. (a) \quad P = xp - C = x\left(100 - \frac{1}{2}x^2\right) - (50x + 37.5) = -\frac{1}{2}x^3 + 50x - 37.50$$

$$P' = -\frac{3}{2}x^2 + 50$$

$$P' = 0 \text{ when } x = \frac{10}{\sqrt{3}} \text{ units and } p = 100 - \frac{1}{2}\left(\frac{10}{\sqrt{3}}\right)^2 = \frac{250}{3} \approx \$83.33.$$

$$(b) \quad \bar{C} = 50 + \frac{37.5}{x}. \text{ When } x = \frac{10}{\sqrt{3}}, \text{ the average price is } \bar{C}\left(\frac{10}{\sqrt{3}}\right) = 50 + \frac{37.5}{(10/\sqrt{3})} \approx \$56.50.$$

18. $P = -\frac{1}{10}s^3 + 6s^2 + 400$

$$P' = -\frac{3}{10}s^2 + 12s = s\left(-\frac{3}{10}s + 12\right)$$

$$P' = 0 \text{ when } s = 0 \text{ or } s = 40.$$

$$P'' = -\frac{3}{5}s + 12$$

$$P''(0) = 12 > 0 \Rightarrow \text{Minimum}$$

$$P''(40) = -12 < 0 \Rightarrow \text{Maximum}$$

$$P'' = -\frac{3}{5}s + 12 = 0 \text{ when } s = 20.$$

The maximum profit occurs when $s = 40$ (or \$40,000) and the point of diminishing returns occurs at $s = 20$ (or \$20,000). The point is (20,000, 2,000,000).

20. Let x = the number of \$40 increases in rent.

$$\text{Rent} = 580 + 40x$$

$$\text{Number of apartments} = 50 - x$$

$$\text{Cost per apartment} = 45$$

$$\text{Profit} = P = (\text{Rent})(\text{Number of apartments}) - (\text{Cost})(\text{Number of apartments})$$

$$= (580 + 40x)(50 - x) - 45(50 - x)$$

$$= -40x^2 + 1465x + 26,750$$

$$P'(x) = -80x + 1465$$

$$P' = 0 \text{ when } x = 18.3125.$$

$$\text{If } x = 18, P = 40,160$$

$$\text{If } x = 19, P = 40,145$$

$$\text{So, } x = 18 \text{ and the rent is } 580 + 40(18) = \$1300.$$

22. $A = ki^2$

$$P = 0.12A - i(A) = ki^2(0.12 - i)$$

$$P' = ki^2(-1) + (0.12 - i)(2ki) = 0.24ki - 3ki^2 = ki(0.24 - 3i)$$

$$P' = 0 \text{ when } i = 0 \text{ and } i = 0.08. \text{ The profit is maximum when } i = 8\%.$$

24. Let k = cost per mile to run the line overland. Then

$$C = 2k\sqrt{x^2 + 1} + k(2 - x) = k[2\sqrt{x^2 + 1} + (2 - x)]$$

$$C' = k\left[\frac{2x}{\sqrt{x^2 + 1}} - 1\right].$$

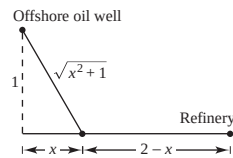
$$C' = 0 \text{ when } \frac{2x}{\sqrt{x^2 + 1}} = 1$$

$$2x = \sqrt{x^2 + 1}$$

$$4x^2 = x^2 + 1$$

$$3x^2 = 1$$

$$x = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3} \text{ mile.}$$



$$26. C = \left(\frac{v^2 + 360}{720} + 8 \right) \left(\frac{110}{v} \right)$$

$$= \frac{110}{720} \left(v + \frac{360}{v} \right) + \frac{880}{v}$$

$$C' = \frac{11}{72} \left(1 - \frac{360}{v^2} \right) - \frac{880}{v^2} = \frac{11}{72} - \frac{935}{v^2}$$

$$C' = 0 \text{ when } 11v^2 = 72.935$$

$$v \approx 78.23 \text{ mph.}$$

$$30. \text{ Since } \frac{dp}{dx} = -\frac{500}{(x+2)^2}, \text{ the price elasticity of demand is}$$

$$\eta = \frac{p/x}{dp/dx} = \frac{500}{x(x+2)} \cdot \frac{(x+2)^2}{-500} = -\frac{x+2}{x}.$$

When $x = 23$, we have

$$\eta = -\frac{23+2}{23} = -\frac{25}{23}.$$

Since $|\eta(23)| = \frac{25}{23} > 1$, the demand is elastic.

$$34. (a) \quad p^3 + x^3 = 9$$

$$3p^2 \frac{dp}{dx} + 3x^2 = 0$$

$$\frac{dp}{dx} = -\frac{x^2}{p^2}$$

$$\eta = \frac{p/x}{-x^2/p^2} = -\frac{p^3}{x^3}$$

$$\text{When } x = 2, p = 1, \text{ and } \eta = -\frac{(1)^3}{(2)^3} = -\frac{1}{8}.$$

$$36. C = 100 \left(\frac{200}{x^2} + \frac{x}{x+30} \right), \quad x \geq 1$$

$$C' = \frac{3000x^3 - 40,000x^2 - 2,400,000x - 36,000,000}{x^5 + 60x^4 + 900x^3}$$

Using the root feature, $C' = 0$ for $x \approx 40.45$

Minimum obtained with order size of 4045.

$$28. \text{ Since } dp/dx = -0.03, \text{ the price elasticity of demand is}$$

$$\eta = \frac{p/x}{dp/dx} = \frac{(5 - 0.03x)/x}{-0.03} = 1 - \frac{5}{0.03x}.$$

When $x = 100$, we have

$$\eta = 1 - \frac{5}{0.03(100)} = -\frac{2}{3}.$$

Since $|\eta(100)| = \frac{2}{3} < 1$, the demand is inelastic.

$$32. \text{ Since } \frac{dp}{dx} = -\frac{0.1}{\sqrt{0.2x}}, \text{ the price elasticity of demand is}$$

$$\begin{aligned} \eta &= \frac{p/x}{dp/dx} = \frac{100 - \sqrt{0.2x}}{x} \cdot \frac{\sqrt{0.2x}}{-0.1} \\ &= 2 - \frac{1000\sqrt{0.2x}}{x}. \end{aligned}$$

When $x = 125$, we have

$$\eta = 2 - \frac{1000\sqrt{0.2(125)}}{125} = -38.$$

Since $|\eta(125)| = 38 > 1$, the demand is elastic.

$$(b) \quad R = xp = x\sqrt[3]{9-x^3}$$

$$\begin{aligned} R' &= x \left[\frac{1}{3}(9-x^3)^{-2/3}(-3x^2) \right] + (9-x^3)^{1/3}(1) \\ &= (9-x^3)^{-2/3}[-x^3 + (9-x^3)] = \frac{9-2x^3}{(9-x^3)^{2/3}} \end{aligned}$$

$$R' = 0 \text{ when } x = \sqrt[3]{9/2} \text{ and } p = \sqrt[3]{9/2}.$$

$$(c) \quad \eta(\sqrt[3]{9/2}) = -\frac{(\sqrt[3]{9/2})^3}{(\sqrt[3]{9/2})^3} = -1$$

$$|\eta(\sqrt[3]{9/2})| = 1$$

$$38. \quad x = 800 - 40p$$

$$p = 20 - \frac{x}{40}$$

$$\frac{dp}{dx} = -\frac{1}{40}$$

When $p = 5$, $x = 600$; $\eta = \frac{p/x}{dp/dx}$. Since $|\eta| < 1$, the demand is inelastic. No, revenue cannot be increased by lowering the price.

40. (a) $S = 201.556t^2 - 502.29t + 2622.8 + \frac{9286}{t}, \quad 4 \leq t \leq 13$

($t = 4$ corresponds to 1994.)

$$S' = 403.112t - 502.29 - \frac{9286}{t^2} > 0$$

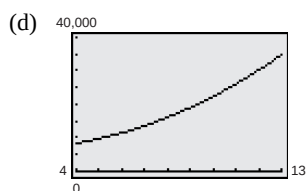
$$S'' = 403.112t + \frac{18,572}{t^3} \neq 0$$

S is increasing most rapidly at $t = 13$ (2003).

(b) S is increasing most slowly at $t = 4$ (1994).

(c) 2003: $S'(13) \approx \$4683$ millions per year per year

1994: $S'(4) \approx \$530$ millions per year per year



42. (a) Demand function

(b) Cost function

(c) Revenue function

(d) Profit function

Section 3.6 Asymptotes

2. A horizontal asymptote occurs at $y = 0$ since $\lim_{x \rightarrow \infty} \frac{4}{(x-2)^3} = 0$ and $\lim_{x \rightarrow -\infty} \frac{4}{(x-2)^3} = 0$.

A vertical asymptote occurs at $x = 2$ since $\lim_{x \rightarrow 2^-} \frac{4}{(x-2)^3} = -\infty$ and $\lim_{x \rightarrow 2^+} \frac{4}{(x-2)^3} = \infty$.

4. A horizontal asymptote occurs at $y = -1$ since $\lim_{x \rightarrow \infty} \frac{2+x}{1-x} = -1$ and $\lim_{x \rightarrow -\infty} \frac{2+x}{1-x} = -1$.

A vertical asymptote occurs at $x = 1$ since $\lim_{x \rightarrow 1^-} \frac{2+x}{1-x} = \infty$ and $\lim_{x \rightarrow 1^+} \frac{2+x}{1-x} = -\infty$.

6. A horizontal asymptote occurs at $y = 0$ since $\lim_{x \rightarrow \infty} \frac{-4x}{x^2+4} = 0$ and $\lim_{x \rightarrow -\infty} \frac{-4x}{x^2+4} = 0$.

The graph has no vertical asymptotes.

8. A horizontal asymptote occurs at $y = 0$ since $\lim_{x \rightarrow \infty} \frac{x^2+1}{x^3-8} = 0$ and $\lim_{x \rightarrow -\infty} \frac{x^2+1}{x^3-8} = 0$.

A vertical asymptote occurs at $x = 2$ since $\lim_{x \rightarrow 2^-} \frac{x^2+1}{x^3-8} = -\infty$ and $\lim_{x \rightarrow 2^+} \frac{x^2+1}{x^3-8} = \infty$.

10. The graph of f has horizontal asymptote at $y = \pm 2$. It has no vertical asymptote and it matches graph (b).
12. The graph of f has a horizontal asymptote at $y = 2$. It has no vertical asymptote and it matches graph (a) since it has a y -intercept at $(0, 2)$.
14. The graph of f has a horizontal asymptote at $y = 2$. It has no vertical asymptote. The y -intercept is $(0, 5)$. Thus, it matches graph (d).

16. $\lim_{x \rightarrow -2^-} \frac{1}{x+2} = -\infty$

18. $\lim_{x \rightarrow 1^+} \frac{2+x}{1-x} = -\infty$

20. $\lim_{x \rightarrow 4} \frac{x^2}{x^2 + 16} = \frac{1}{2}$

22. $\lim_{x \rightarrow 0^-} \left(x^2 - \frac{1}{x} \right) = 0 - (-\infty) = \infty$

24. $\lim_{x \rightarrow \infty} \frac{5x^3 + 1}{10x^3 - 3x^2 + 7} = \frac{5}{10} = \frac{1}{2}$

26. $\lim_{x \rightarrow \infty} \frac{2x^{10} - 1}{10x^{11} - 3} = 0$

28. $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 3x + 1}{x^2 - 3x + 2} = \infty$

30. $\lim_{x \rightarrow \infty} \left(2 - \frac{1}{x^3} \right) = 2 - 0 = 2$

32. $\lim_{x \rightarrow \infty} \left(\frac{2x^2}{x-1} + \frac{3x}{x+1} \right) = \infty + 3 = \infty$

34.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1	5.132	50.126	500.125	5000.126	50,000.2	500,000

$$\lim_{x \rightarrow \infty} [x^2 - x\sqrt{x(x-1)}] = \infty$$

36. $f(x) = \frac{3x^2}{0.1x^2 + 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	2.7273	27.2727	29.9700	29.9997	30	30	30

$$\lim_{x \rightarrow \infty} f(x) = 30$$

38.

x	-10^6	-10^4	-10^2	10^0	10^2	10^4	10^6
$f(x)$	-2,000,000.5	-20,000.5	-200.5	1	0.501	0.500	0.500

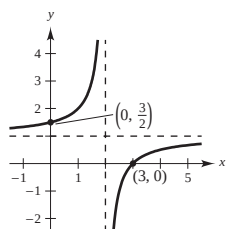
$$\lim_{x \rightarrow \infty} [x - \sqrt{x(x-1)}] = 0.5, \quad \lim_{x \rightarrow -\infty} [x - \sqrt{x(x-1)}] = -\infty$$

40. Intercepts: $(3, 0)$, $(0, \frac{3}{2})$

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 2$

No relative extrema



42. $f(x) = \frac{x}{x^2 + 4}$

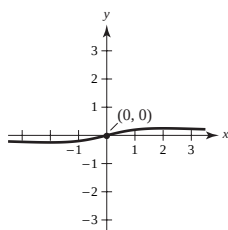
$$f'(x) = \frac{4 - x^2}{(x^2 + 4)^2} = 0 \Rightarrow x = \pm 2$$

Intercept: $(0, 0)$

Relative maximum: $(2, \frac{1}{4})$

Relative minimum: $(-2, -\frac{1}{4})$

Horizontal asymptote: $y = 0$



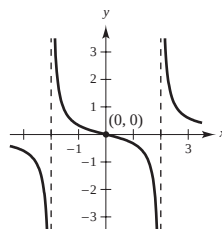
44. Intercept: $(0, 0)$

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 2$

$$g'(x) = \frac{-x^2 - 4}{(x^2 - 4)^2}$$

No relative extrema



46. $y = \frac{4}{x^2}$

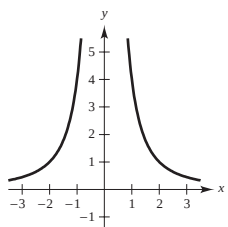
No intercepts

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = 0$

$$y' = -\frac{8}{x^3}$$

No relative extrema



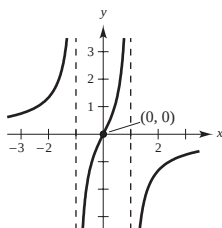
48. Intercept: $(0, 0)$

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 1$

$$y' = -\frac{2(x^2 + 1)}{(1 - x^2)^2}$$

No relative extrema



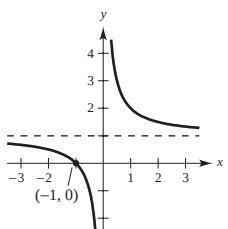
50. Intercept: $(-1, 0)$

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 0$

$$y' = -\frac{1}{x^2}$$

No relative extrema



52. $f(x) = \frac{x - 2}{(x - 1)(x - 3)}$

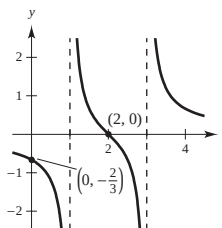
Intercepts: $(2, 0)$, $(0, -2/3)$

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = 1$ and $x = 3$

$$f'(x) = \frac{-(x^2 - 4x + 5)}{(x^2 - 4x + 3)^2}$$

No relative extrema

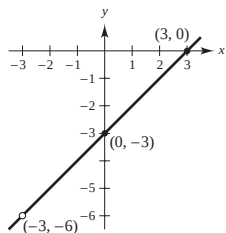


$$54. g(x) = \frac{x^2 - 9}{x + 3} = \begin{cases} x - 3, & x \neq -3 \\ \text{undefined}, & x = -3 \end{cases}$$

Intercept: (3, 0), (0, -3)

No asymptotes

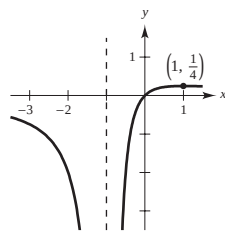
No relative extrema



56. Intercept: (0, 0)

Horizontal asymptote: $y = 0$ Vertical asymptote: $x = -1$

$$y' = \frac{1 - x}{(x + 1)^3}$$

Relative maximum: $(1, \frac{1}{4})$ 

$$58. (a) \bar{C} = 0.5 + \frac{500}{x}$$

$$(b) \bar{C}(750) = \$7.50$$

$$\bar{C}(1750) = \$0.90$$

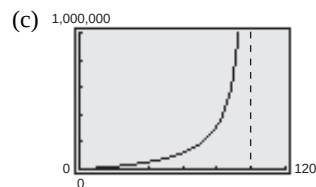
$$(c) \lim_{x \rightarrow \infty} \bar{C} = \lim_{x \rightarrow \infty} \left(0.5 + \frac{500}{x} \right) = \$0.50$$

$$60. (a) C(15) \approx \$14,117.65$$

$$C(50) = \$80,000$$

$$C(90) = \$720,000$$

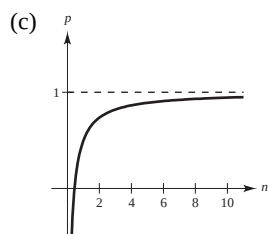
$$(b) \lim_{x \rightarrow 100^-} \frac{80,000}{100 - p} = \infty$$



62. (a)

n	1	2	3	4	5	6	7	8	9	10
P	0.50	0.74	0.82	0.86	0.89	0.91	0.92	0.93	0.94	0.95

$$(b) \lim_{n \rightarrow \infty} \frac{0.5 + 0.9(n - 1)}{1 + 0.9(n - 1)} = 1$$



$$64. (a) \bar{P} = \frac{R - C}{x} = \frac{69.9x - (34.5x + 15,000)}{x} = \frac{35.4x - 15,000}{x} = 35.4 - \frac{15,000}{x}$$

$$(b) \bar{P}(1000) = \$20.40$$

$$\bar{P}(10,000) = \$33.90$$

$$\bar{P}(100,000) = \$35.25$$

$$(c) \lim_{x \rightarrow \infty} \left(35.4 - \frac{15,000}{x} \right) = \$35.40$$

Section 3.7 Curve Sketching: A Summary

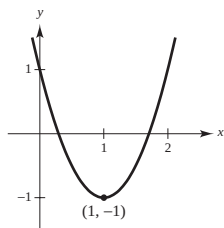
2. $y = 2x^2 - 4x + 1$

$$y' = 4x - 4 = 4(x - 1)$$

$$y'' = 4$$

Intercepts: $(0, 1), (1 \pm \sqrt{2}/2, 0)$ Relative minimum: $(1, -1)$

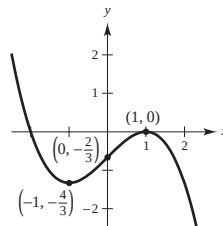
Concave upward



4. $y = -\frac{1}{3}(x^3 - 3x + 2)$

$$y' = -\frac{1}{3}(3x^2 - 3) = 1 - x^2$$

$$y'' = -2x$$

Intercepts: $(0, -\frac{2}{3}), (1, 0), (-2, 0)$ Relative minimum: $(-1, -\frac{4}{3})$ Relative maximum: $(1, 0)$ Point of inflection: $(0, -\frac{2}{3})$ 

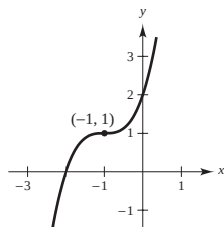
6. $y = x^3 + 3x^2 + 3x + 2$

$$y' = 3x^2 + 6x + 3 = 3(x + 1)^2$$

$$y'' = 6x + 6 = 6(x + 1)$$

Intercepts: $(0, 2), (-2, 0)$

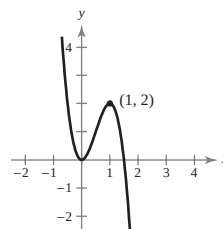
No relative extrema

Point of inflection: $(-1, 1)$ 

8. $y = -4x^3 + 6x^2$

$$y' = -12x^2 + 12x = 12x(1 - x)$$

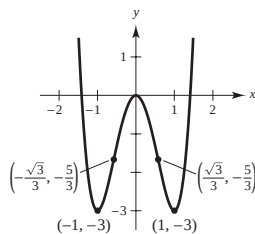
$$y'' = -24x + 12 = 12(1 - 2x)$$

Critical numbers: $x = 0, 1$ Relative maximum: $(1, 2)$ Relative minimum: $(0, 0)$ Point of inflection: $(\frac{1}{2}, 1)$ 

10. $y = 3x^4 - 6x^2 = 3x^2(x^2 - 2)$

$$y' = 12x^3 - 12x = 12x(x^2 - 1)$$

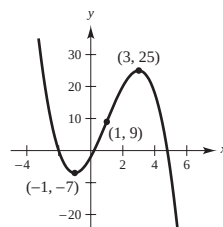
$$y'' = 36x^2 - 12 = 12(3x^2 - 1)$$

Intercepts: $(0, 0), (\pm\sqrt{2}, 0)$ Relative maximum: $(0, 0)$ Relative minima: $(\pm 1, -3)$ Points of inflection: $(\pm\sqrt{3}/3, -5/3)$ 

12. $f(x) = -x^3 + 3x^2 + 9x - 2$

$$f'(x) = -3x^2 + 6x + 9 = -3(x + 1)(x - 3)$$

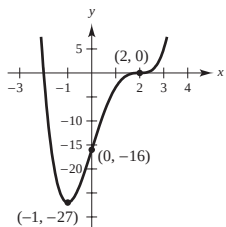
$$f''(x) = -6x + 6 = -6(x - 1)$$

Relative maximum: $(3, 25)$ Relative minimum: $(-1, -7)$ Point of inflection: $(1, 9)$ 

14. $f(x) = x^4 - 4x^3 + 16x - 16$

$$f'(x) = 4x^3 - 12x^2 + 16 = 4(x+1)(x-2)^2$$

$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

Relative minimum: $(-1, -27)$ Points of inflection: $(0, -16), (2, 0)$ 

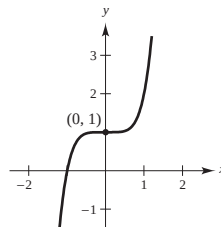
16. $f(x) = x^5 + 1$

$$f'(x) = 5x^4$$

$$f''(x) = 20x^3$$

Intercepts: $(0, 1), (-1, 0)$

No relative extrema

Point of inflection: $(0, 1)$ 

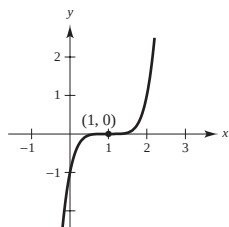
18. $y = (x-1)^5$

$$y' = 5(x-1)^4$$

$$y'' = 20(x-1)^3$$

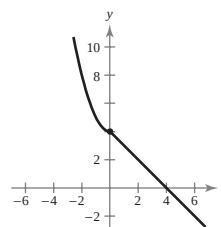
Intercepts: $(0, -1), (1, 0)$

No relative extrema

Point of inflection: $(1, 0)$ 

20. $y = \begin{cases} x^2 + 4, & x < 0 \\ 4 - x, & x \geq 0 \end{cases}$

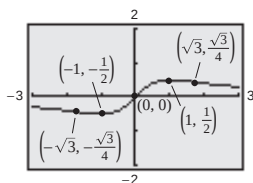
$$y' = \begin{cases} 2x, & x < 0 \\ -1, & x > 0 \end{cases}$$



22. $y = \frac{x}{x^2 + 1}$

$$y' = \frac{1 - x^2}{(x^2 + 1)^2}$$

$$y'' = \frac{2x(x^2 - 3)}{(x^2 + 1)^3}$$

Intercept: $(0, 0)$ Relative maximum: $(1, 1/2)$ Relative minimum: $(-1, -1/2)$ Points of inflection: $(0, 0), (-\sqrt{3}, -\sqrt{3}/4), (\sqrt{3}, \sqrt{3}/4)$ 

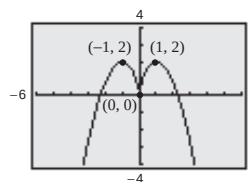
24. $y = 3x^{2/3} - x^2 = x^{2/3}(3 - x^{4/3})$

$$y' = 2x^{-1/3} - 2x = \frac{2}{\sqrt[3]{x}} - 2x = \frac{2(1 - x^{4/3})}{\sqrt[3]{x}}$$

$$y'' = -\frac{2}{3}x^{-4/3} - 2 = -\frac{2}{3x^{4/3}} - 2$$

Intercepts: $(0, 0), (\pm \sqrt[3]{27}, 0)$ Relative maxima: $(-1, 2), (1, 2)$ Relative minimum: $(0, 0)$

Concave downward



26. $y = (1 - x)^{2/3}$

$$y' = -\frac{2}{3}(1 - x)^{-1/3}$$

$$= -\frac{2}{3\sqrt[3]{1 - x}}$$

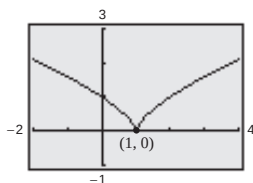
$$y'' = -\frac{2}{9}(1 - x)^{-4/3}$$

$$= -\frac{2}{9(1 - x)^{4/3}}$$

Intercepts: (0, 1), (1, 0)

Relative minimum: (1, 0)

Concave downward



28. $y = x^{-1/3}$

$$y' = -\frac{1}{3}x^{-4/3}$$

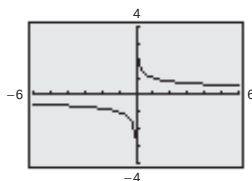
$$y'' = \frac{4}{9}x^{-7/3}$$

No intercepts

No relative extrema

Decreasing

No points of inflection

Concave downward on $(-\infty, 0)$ Concave upward on $(0, \infty)$ 

30. $y = x^{4/3}$

$$y' = \frac{4}{3}x^{1/3}$$

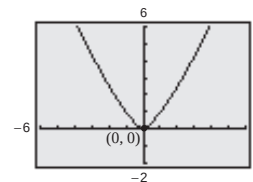
$$y'' = \frac{4}{9}x^{-2/3}$$

Intercept: (0, 0)

Relative minimum: (0, 0)

No points of inflection

Concave upward



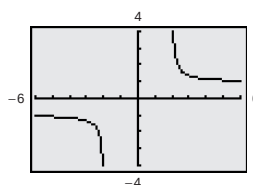
32. $y = \frac{x}{\sqrt{x^2 - 4}}$

Domain: $|x| > 2$

$$y' = \frac{-4}{(x^2 - 4)^{3/2}} \text{ Decreasing on } (-\infty, 2) \text{ and } (2, \infty).$$

No relative extrema.

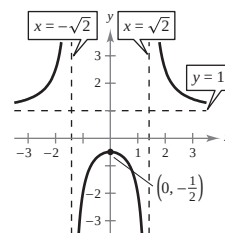
$$y'' = \frac{12x}{(x^2 - 4)^{5/2}} \text{ No points of inflection.}$$

Vertical asymptote at $x = \pm 2$.Horizontal asymptote: $y = 1$ for $x > 2$ $y = -1$ for $x < -2$ 

34. $y = \frac{x^2 + 1}{x^2 - 2}$

$$y' = -\frac{6x}{(x^2 - 2)^2}$$

$$y'' = \frac{6(3x^2 + 2)}{(x^2 - 2)^3}$$

Intercepts: $(0, -1/2)$ Relative maximum: $(0, -1/2)$ Vertical asymptotes: $x = \pm\sqrt{2}$ Horizontal asymptote: $y = 1$ Domain: $(-\infty, -\sqrt{2}), (-\sqrt{2}, \sqrt{2}), (\sqrt{2}, \infty)$ 

36. $y = \frac{x^2 - 6x + 12}{x - 4}$

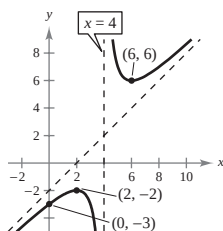
$$y' = \frac{x^2 - 8x + 12}{(x - 4)^2} = \frac{(x - 2)(x - 6)}{(x - 4)^2}$$

$$y'' = \frac{8}{(x - 4)^3}$$

Intercept: (0, -3)

Relative maximum: (6, 6)

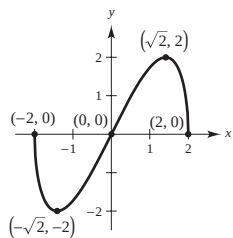
Relative minimum: (2, -2)

Vertical asymptote: $x = 4$ Domain: $(-\infty, 4), (4, \infty)$ 

38. $y = x\sqrt{4 - x^2}$

$$y' = \frac{2(2 - x^2)}{\sqrt{4 - x^2}}$$

$$y'' = \frac{2x(x^2 - 6)}{(4 - x^2)^{3/2}}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$ Relative maximum: $(\sqrt{2}, 2)$ Relative minimum: $(-\sqrt{2}, -2)$ Point of inflection: $(0, 0)$ Domain: $[-2, 2]$ 

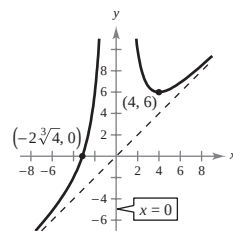
40. $y = x + \frac{32}{x^2}$

$$y' = 1 - \frac{64}{x^3} = \frac{x^3 - 64}{x^3}$$

$$y'' = \frac{192}{x^4}$$

Intercept: $(-2\sqrt[3]{4}, 0)$ Relative minimum: $(4, 6)$

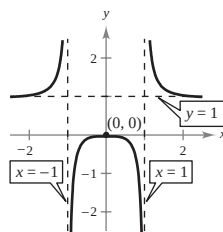
Concave upward

Vertical asymptote: $x = 0$ Domain: $(-\infty, 0), (0, \infty)$ 

42. $y = x^4/(x^4 - 1)$

$$y' = -4x^3/(x^4 - 1)^2$$

$$y'' = 4x^2(5x^4 + 3)/(x^4 - 1)^3$$

Intercept: $(0, 0)$ Horizontal asymptote: $y = 1$ Vertical asymptotes: $x = \pm 1$ Domain: $(-\infty, -1), (-1, 1), (1, \infty)$ Relative maximum: $(0, 0)$ 44. Since $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow -\infty} f(x) = -\infty$, we have $a > 0$.

$$f(x) = x^3 + x^2 - x + 1$$

(Solution not unique)

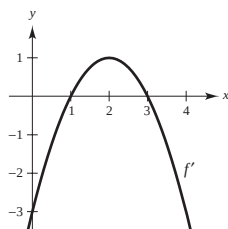
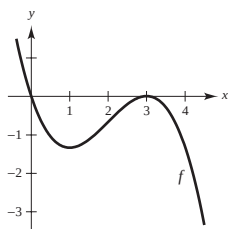
46. Since $\lim_{x \rightarrow \infty} f(x) = -\infty$ and $\lim_{x \rightarrow -\infty} f(x) = \infty$, we have $a < 0$.

$$f(x) = -x^3 + 1$$

(Solution not unique)

48. $f'(x) = -(x - 1)(x - 3) = -x^2 + 4x - 3$

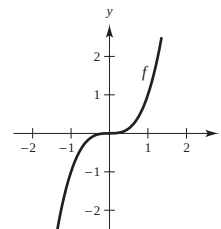
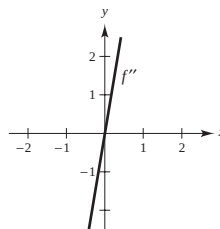
$$f(x) = -\frac{1}{3}x^3 + 2x^2 - 3x$$



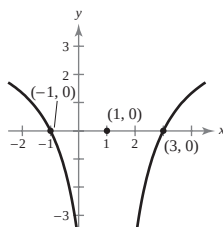
50. $f''(x) = 6x$

$$f' = 3x^2$$

$$f(x) = x^3$$



$$52. f(x) = \begin{cases} \frac{4(x+1)}{x-1}, & x < 1 \\ 0, & x = 1 \\ \frac{4(x-3)}{x-1}, & x > 1 \end{cases}$$

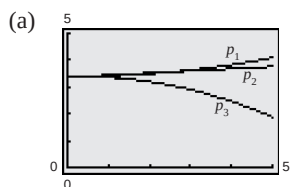


$$54. \text{ Model I: } p_1 = 0.03t^2 - 0.01t + 3.39$$

$$\text{Model II: } p_2 = 0.08t + 3.36$$

$$\text{Model III: } p_3 = -0.07t^2 + 0.05t + 3.38$$

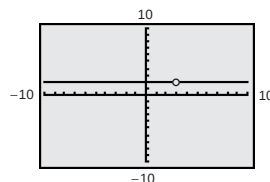
$t = 0$ corresponds to 2001, $0 \leq t \leq 5$.



(b) Models I and II

(c) I most optimistic, III most pessimistic

$$56. h(x) = \frac{6-2x}{3-x} = 2, \quad x \neq 3$$



Section 3.8 Differentials and Marginal Analysis

$$2. dy = 3x^{1/2}dx = 3\sqrt{x} dx$$

$$4. dy = 4(1-2x^2)^3(-4x)dx = -16x(1-2x^2)^3 dx$$

$$6. dy = \frac{1}{3}(6x^2)^{-2/3}(12x) dx = \frac{4x}{\sqrt[3]{36x^4}} dx = \frac{4}{\sqrt[3]{36x}} dx$$

$$8. f(x) = \sqrt{3x}, \quad x = 1, \quad \Delta x = 0.01$$

$$\Delta y = f(x + \Delta x) - f(x) = \sqrt{3(1.01)} - \sqrt{3(1)} \approx 0.0086387$$

$$10. f(x) = \frac{x}{x^2 + 1}, \quad x = 1, \quad \Delta x = 0.01$$

$$\Delta y = f(x + \Delta x) - f(x) = \frac{1.01}{(1.01)^2 + 1} - \frac{1}{1^2 + 1} \approx -0.0000247512$$

$$12. y = 1 - 2x^2, \quad x = 0, \quad \Delta x = dx = -0.1$$

$$dy = -4x dx = 0$$

$$\Delta y = 1 - 2(-0.1)^2 - 1 = -0.02$$

$$14. y = 2x^3 + 1, \quad x = 2, \quad \Delta x = dx = 0.01$$

$$dy = 6x^2 dx = 6(2^2)(0.01) = 0.24$$

$$\Delta y = [2(2.01)^3 + 1] - [2(2)^3 + 1] = 0.241202$$

16. $dy = 5x^4 dx$

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\frac{dy}{\Delta y}$
1.0000	80.0000	211.0000	131.0000	0.3791
0.5000	40.0000	65.6562	25.6562	0.6092
0.1000	8.0000	8.8410	0.8410	0.9049
0.0100	0.8000	0.8080	0.0080	0.9901
0.0010	0.0800	0.0801	0.0001	0.9990

18. $y = \frac{1}{x}, x = 2, dy = \frac{-1}{x^2} dx, \Delta y = \frac{1}{x + \Delta x} - \frac{1}{x} = \frac{-\Delta x}{x(x + \Delta x)}$

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\frac{dy}{\Delta y}$
1.0	-0.25	-0.1667	0.0833	1.5
0.5	-0.125	-0.1	0.025	1.25
0.1	-0.025	-0.0238	0.00119	1.05
0.01	-0.0025	-0.00249	0.00001	1.005
0.001	-0.00025	-0.00025	0.0	1.0005

20. $dy = \frac{1}{2\sqrt{x}} dx, \Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\frac{dy}{\Delta y}$
1.000	0.3536	0.3178	-0.0358	1.1126
0.500	0.1768	0.1669	-0.0099	1.0590
0.100	0.0354	0.0349	-0.0005	1.0123
0.010	0.0035	0.0035	0.0000	1.0012
0.001	0.0004	0.0004	0.0000	1.0001

22. $f(x) = 3x^2 - 1, (2, 11)$

$f'(x) = 6x, f'(2) = 12$

$y - 11 = 12(x - 2)$

$y = 12x - 13$

$f(2 + 0.01) = 11.1203$

$y(2 + 0.01) = 11.12$

$f(2 - 0.01) = 10.8803$

$y(2 - 0.01) = 10.88$

24. $f(x) = \sqrt{25 - x^2}$

$f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}$

$y - 4 = \frac{-3}{\sqrt{25 - 9}}(x - 3)$

$y - 4 = -\frac{3}{4}(x - 3)$

$y = -\frac{3}{4}x + \frac{25}{4}$

$f(3 + 0.01) = 3.99248$

$y(3 + 0.01) = 3.9925$

$f(3 - 0.01) = 4.00748$

$y(3 - 0.01) = 4.0075$

$$26. N = \frac{10(5 + 3t)}{1 + 0.04t}$$

$$dN = \frac{(1 + 0.04t)(30) - 10(5 + 3t)(0.04)}{(1 + 0.04t)^2} dt = \frac{28}{(1 + 0.04t)^2} dt$$

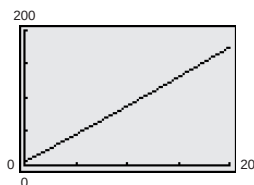
When $t = 5$ and $dt = 6 - 5 = 1$, we have the following.

$$dN = \frac{28}{[1 + 0.04(5)]^2} (1) = \frac{28}{1.44} \approx 19.44$$

The change in herd size will be approximately 19 deer.

$$28. x = 10, dx = \Delta x = 1$$

$$\begin{aligned} \Delta C &\approx dC = (0.05x + 8) dx \\ &= (0.05(10) + 8)(1) \\ &= 8.5 = \$8.50 \end{aligned}$$



Checking, $C(11) - C(10) = 96.025 - 87.5 = 8.525$

$$30. x = 15, dx = \Delta x = 1$$

$$\begin{aligned} \Delta R &\approx dR = (50 - 3x) dx \\ &= (50 - 3(15))(1) \\ &= 5 \Rightarrow \$5.00 \end{aligned}$$



Checking, $R(16) - R(15) = 416 - 412.5 = 3.8$

$$32. P = -x^2 + 60x - 100, \quad x = 25, \quad dx = 1$$

$$dP = (-2x + 60) dx = [-2(25) + 60](1) = \$10$$

$$34. x = \frac{k}{p^2}$$

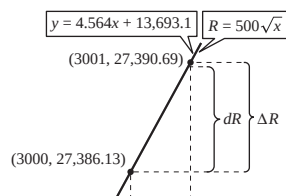
$$2500 = \frac{k}{100}$$

$$250,000 = k$$

$$x = \frac{250,000}{p^2} \Rightarrow p^2 = \frac{250,000}{x} \Rightarrow p = \frac{500}{\sqrt{x}}$$

$$R = xp = x \left(\frac{500}{\sqrt{x}} \right) = 500\sqrt{x}$$

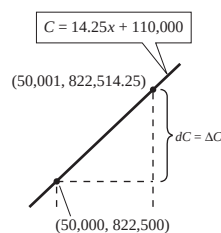
$$dR = \frac{250}{\sqrt{x}} dx = \frac{250}{\sqrt{3000}} (1) \approx \$4.56$$



$$36. C = 14.25x + 110,000$$

$$dC = 14.25 dx = 14.25(1) = \$14.25$$

$\Delta C = dC$ because C is a linear function.



38. $A = x^2$

$$dA = 2x \, dx$$

When $x = 12$ and $dx = \pm 1/64$, we have

$$dA = 2(12)\left(\pm \frac{1}{64}\right) = \pm \frac{3}{8} \text{ in}^2.$$

When $A = 144$, the relative error is

$$\frac{dA}{A} = \frac{\pm 3/8}{144} \approx 0.0026.$$

42. $C = \frac{3t}{27 + t^3}$

$$dC = \frac{3(27 - 2t^3)}{(27 + t^3)^2} dt$$

When $t = 1$ and $dt = \frac{1}{2}$, we have

$$dC = \frac{3(25)}{(28)^2}\left(\frac{1}{2}\right) \approx 0.0478.$$

44. True. $\Delta y = [a(x + \Delta x) + b] - [ax + b] = a \Delta x \Rightarrow \frac{\Delta y}{\Delta x} = a = \frac{dy}{dx}$

Review Exercises for Chapter 3

2. $g(x) = (x - 1)^2(x - 3)$

$$g'(x) = (x - 1)(3x - 7)$$

Critical numbers: $x = 1, x = \frac{7}{3}$

6. $g(x) = -x^2 + 7x - 12$

$$g'(x) = -2x + 7$$

Critical number: $x = \frac{7}{2}$ Increasing on $(-\infty, \frac{7}{2})$ Decreasing on $(\frac{7}{2}, \infty)$

10. (a) $S = 5.8583t^2 - 28.943t^2 - 34.36t + 940.6$,
 $-2 \leq t \leq 2$, $t = 0$ corresponds to 2000.

$$S' = 17.5749t^2 - 57.886t - 34.36$$

 S increasing on $(-2, -0.51)$

(c) Shipments increasing from 1998 to mid-1999.

Shipments decreasing from mid-1999 to 2002.

40. $V = x^3$, $A = 6x^2$

(a) $dV = 3x^2 dx$

$$\frac{dV}{V} = \frac{3x^2 dx}{x^3} = \frac{3 dx}{x}$$

When $x = 12$ and $dx = \pm 0.03$, we have

$$dV = 3(12)^2(\pm 0.03) = \pm 12.96 \text{ in}^3.$$

$$\frac{dV}{V} = \frac{3(\pm 0.03)}{12} = \pm 0.0075$$

(b) $dA = 12x \, dx$

$$\frac{dA}{A} = \frac{12x \, dx}{6x^2} = \frac{2 \, dx}{x}$$

$$dA = 12(12)(\pm 0.03) = \pm 4.32 \text{ in}^2$$

$$\frac{dA}{A} = \frac{2(\pm 0.03)}{12} = \pm 0.005$$

4. $f(x) = (x + 1)^3$

$$f'(x) = 3(x + 1)^2$$

Critical number: $x = -1$

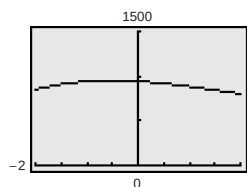
8. $f(x) = -x^3 + 6x^2 - 2$

$$f'(x) = -3x^2 + 12x = -3x(x - 4)$$

Critical numbers: $x = 0, 4$ Increasing on $(0, 4)$ Decreasing on $(-\infty, 0), (4, \infty)$

(b) S decreasing on $(-0.51, 2)$

(d)



12. $f(x) = \frac{1}{4}x^4 - 8x$

$f'(x) = x^3 - 8$

Critical number: $x = 2$

Intervals	$(-\infty, 2)$	$(2, \infty)$
Sign of $f'(x)$	-	+
Conclusion	Decreasing	Increasing

Relative minimum: $(2, -12)$

16. $s(x) = x^4 - 8x^2 + 3$

$s'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x - 2)(x + 2)$

Critical numbers: $x = 0, 2, -2$

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Sign of f'	-	+	-	+
Conclusion	Decreasing	Increasing	Decreasing	Increasing

Relative minima: $(-2, -13), (2, -13)$ Relative maximum: $(0, 3)$

18. $f(x) = \frac{2}{x^2 - 1}$

$f'(x) = \frac{-4x}{(x^2 - 1)^2}$

Critical number: $x = 0$ Discontinuities: $x = \pm 1$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of f'	+	+	-	-
Conclusion	Increasing	Increasing	Decreasing	Decreasing

Relative maximum: $(0, -2)$

20. $g(x) = x - 6\sqrt{x}, \quad x > 0$

$g'(x) = \frac{1-3}{\sqrt{x}} = \frac{\sqrt{x}-3}{\sqrt{x}}$

Critical number: $x = 9$ Increasing on $(9, \infty)$ Decreasing on $(0, 9)$ Relative minimum: $(9, -9)$

22. $f(x) = x^4 - 2x^3, \quad [0, 2]$

$f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$

Critical numbers: $x = 0, \frac{3}{2}$

x	$f(x)$	
0	0	maximum
$\frac{3}{2}$	-1.6875	minimum
2	0	maximum

24. $f(x) = x^3 + 2x^2 - 3x + 4, \quad [-3, 2]$

$$f'(x) = 3x^2 + 4x - 3$$

$$\text{Critical numbers: } x = \frac{-4 \pm \sqrt{52}}{6} = \frac{-2 \pm \sqrt{13}}{3}$$

x	$f(x)$	
-3	4	
$(-2 - \sqrt{13})/3$	10.0646	
$(-2 + \sqrt{13})/3$	3.1206	minimum
2	14	maximum

28. $f(x) = -x^4 + x^2 + 2, \quad [0, 2]$

$$f'(x) = -4x^3 + 2x = 2x(1 - 2x^2)$$

$$\text{Critical numbers: } x = 0, \frac{\sqrt{2}}{2}$$

x	$f(x)$	
0	2	
$\frac{\sqrt{2}}{2}$	-10	minimum
2	$\frac{9}{4}$	maximum

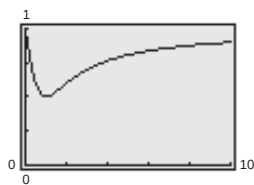
32. $O = \frac{t^2 - t + 1}{t^2 + 1}, \quad 0 \leq t$

The graph of O is:

(a) Lowest level is $\frac{1}{2}$ at $t = 1$.

(b) Highest level is 1 at $t = 0$.

(c) As $t \rightarrow \infty, O \rightarrow \frac{1}{2}$.



36. $h(x) = x^3 - 6x$

$$h'(x) = 3x^2 - 6$$

$$h''(x) = 6x$$

$h''(x) > 0$ for $x > 0$: concave upward on $(0, \infty)$

$h''(x) < 0$ for $x < 0$: concave downward on $(-\infty, 0)$

26. $f(x) = 2\sqrt{x} - x, \quad [0, 9]$

$$f'(x) = \frac{1}{\sqrt{x}} - 1$$

$$\text{Critical number: } x = 1$$

x	$f(x)$	
0	0	
1	1	maximum
9	-3	minimum

30. $f(x) = \frac{8}{x} + x, \quad [1, 4]$

$$f'(x) = \frac{-8}{x^2} + 1 = \frac{x^2 - 8}{x^2}$$

$$\text{Critical number: } x = 2\sqrt{2}$$

x	$f(x)$	
1	9	maximum
$2\sqrt{2}$	6	minimum
4	$4\sqrt{2}$	

34. $h(x) = x^5 - 10x^2$

$$h'(x) = 5x^4 - 20x$$

$$h''(x) = 20x^3 - 20 = 20(x - 1)(x^2 + x + 1)$$

$f''(x) > 0$ for $x > 1$: concave upward on $(1, \infty)$

$f''(x) < 0$ for $x < 1$: concave downward on $(-\infty, 1)$

38. $f(x) = \frac{1}{4}x^4 - 2x^2 - x$

$$f'(x) = x^3 - 4x - 1$$

$$f''(x) = 3x^2 - 4$$

$$f''(x) = 0 \text{ when } x = \pm \frac{2}{\sqrt{3}}$$

Since f changes concavity at $x = \pm 2/\sqrt{3}$, the points of inflection are $(-2/\sqrt{3}, (-20/9) + (2/\sqrt{3}))$ and $(2/\sqrt{3}, (-20/9) - (2/\sqrt{3}))$.

40. $f(x) = (x + 2)^2(x - 4) = x^3 - 12x - 16$

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x = 0 \text{ when } x = 0.$$

Since f changes concavity at $x = 0$, $(0, -16)$ is a point of inflection.

44. $f(x) = (x - 2)^2(x + 2)^2 = x^4 - 8x^2 + 16$

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$$

$$f''(x) = 12x^2 - 16$$

Critical numbers: $x = 0, \pm 2$

$f''(0) < 0 \Rightarrow (0, 16)$ is a relative maximum.

$f''(\pm 2) > 0 \Rightarrow (2, 0)$ and $(-2, 0)$ are relative minima.

42. $f(x) = x(x^2 - 3x - 9) = x^3 - 3x^2 - 9x$

$$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x - 3)(x + 1)$$

$$f''(x) = 6x - 6$$

Critical numbers: $x = 3, -1$

$f''(3) > 0 \Rightarrow (3, -27)$ is a relative minimum.

$f''(-1) < 0 \Rightarrow (-1, 5)$ is a relative maximum.

46. $R = -\frac{2}{3}(x^3 - 12x^2 - 6), \quad 0 \leq x \leq 8$

$$R' = -\frac{2}{3}(3x^2 - 24x)$$

$$R'' = -\frac{2}{3}(6x - 24)$$

$R'' = 0$ when $x = 4$.

The point of diminishing return is $\left(4, \frac{268}{3}\right)$.

48. Let x and y be the lengths shown in the figure. By similar triangles we have

$$\frac{5}{x} = \frac{y}{x + 4}.$$

Thus, $y = 5 + (20/x)$. To minimize the length of the hypotenuse, we have the following.

$$z = \sqrt{(x + 4)^2 + y^2} = \sqrt{(x + 4)^2 + \left(5 + \frac{20}{x}\right)^2} = \sqrt{x^2 + 8x + 41 + \frac{200}{x} + \frac{400}{x^2}}$$

$$\frac{dz}{dx} = \frac{1}{2} \left[(x + 4)^2 + \left(5 + \frac{20}{x}\right)^2 \right]^{-1/2} \left(2x + 8 - \frac{200}{x^2} - \frac{800}{x^3} \right)$$

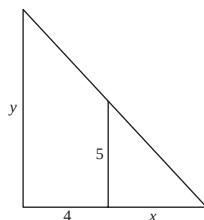
$dz/dx = 0$ when

$$x + 4 - \frac{100}{x^2} - \frac{400}{x^3} = 0$$

$$x^4 + 4x^3 - 100x - 400 = 0$$

$$(x + 4)(x^3 - 100) = 0.$$

When $x = \sqrt[3]{100}$, $z \approx 12.7$ feet.



50. (a) $xy = 4800$

$$\text{Cost} = C = 3(x + 2y) + 4(x) = 7x + 6y = 7x + 6\left(\frac{4800}{x}\right)$$

$$C' = 7 - \frac{28,800}{x^2} = 0 \Rightarrow x \approx 64.14 \text{ feet}$$

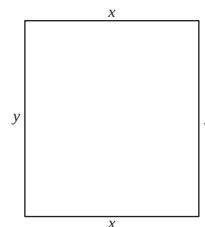
$$y \approx 74.83 \text{ feet}$$

(b) If costs increase by \$1, then

$$C = 4(x + 2y) + 5(x) = 9x + 8y = 9x + 8\left(\frac{4800}{x}\right)$$

$$C' = 9 - \frac{38,400}{x^2} = 0 \Rightarrow x \approx 65.32 \text{ feet}$$

$$y \approx 73.48 \text{ feet.}$$



52. (a) $N = -2.870t^3 + 79.62t^2 - 639.1t + 3473$, $6 \leq t \leq 11$, $t = 6$ corresponds to 1996.

$$N'(t) = -8.61t^2 + 159.24t - 639.1$$

$$N''(t) = -17.22t + 159.24$$

$$N''(t) = 0 \text{ for } t \approx 9.25$$

$$\frac{dN}{dt} \text{ increasing on } 6 < t < 9.25, \text{ decreasing on } 9.25 < t < 11.$$

- (b) $\lim_{t \rightarrow \infty} N = -\infty$ because the cubic term has a negative coefficient.

- (c) Answers will vary.

54. (a) $P = R - C = xp - C = x(36 - 4x) - (2x^2 + 6)$

$$= -6x^2 + 36x - 6$$

$$P'(x) = -12x + 36 = 0 \Rightarrow x = 3 \text{ for maximum profit.}$$

(b) $\bar{C} = \frac{C}{x} = \frac{2x^2 + 6}{x} = 2x + \frac{6}{x}$

$$\bar{C}' = 2 - \frac{6}{x^2} = 0 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3} \approx 1.73 \text{ units}$$

56. $F = ks^{3/2}$

When $s = 25$, $F = 50$ we have

$$50 = k(25)^{3/2} \Rightarrow k = \frac{2}{5}$$

$$F = \frac{2}{5}s^{3/2}.$$

The total cost per mile is

$$C = \frac{F}{s} + \frac{100}{s} = \frac{2}{5}s^{1/2} + \frac{100}{s}$$

$$\frac{dC}{ds} = \frac{1}{5}s^{-1/2} - \frac{100}{s^2}.$$

$$\frac{dC}{ds} = 0 \text{ when } s = 500^{2/3} \approx 63 \text{ miles per hour.}$$

58. $P = xp - C - xt$

$$t = \$5:$$

$$P = x(600 - 3x) - (0.3x^2 + 6x + 600) - 5x$$

$$= -3.3x^2 + 589x - 600$$

$$\frac{dP}{dx} = -6.6x + 589$$

$$\frac{dP}{dx} = 0 \text{ when } x = \frac{589}{6.6}.$$

$$P\left(\frac{589}{6.6}\right) \approx \$25,681.89$$

$$t = \$20:$$

$$P = x(600 - 3x) - (0.3x^2 + 6x + 600) - 20x$$

$$= -3.3x^2 + 574x - 600$$

$$\frac{dP}{dx} = -6.6x + 574$$

$$\frac{dP}{dx} = 0 \text{ when } x = \frac{574}{6.6}.$$

$$P\left(\frac{574}{6.6}\right) \approx \$24,360.30$$

$$t = \$10:$$

$$P = x(600 - 3x) - (0.3x^2 + 6x + 600) - 10x$$

$$= -3.3x^2 + 584x - 600$$

$$\frac{dP}{dx} = -6.6x + 584$$

$$\frac{dP}{dx} = 0 \text{ when } x = \frac{584}{6.6}.$$

$$P\left(\frac{584}{6.6}\right) \approx \$25,237.58$$

60. $p = 60 - 0.04x$, $0 \leq x \leq 1500$, $\frac{dp}{dx} = -0.04$

$$\eta = \frac{p/x}{dp/dx} = \frac{(60 - 0.04x)/x}{-0.04} = \frac{x - 1500}{x}$$

$$|\eta| = 1 = \left| \frac{x - 1500}{x} \right| \Rightarrow x = |x - 1500| \Rightarrow x = 750$$

For $0 < x < 750$, $|\eta| > 1$, elastic.

For $750 < x < 1500$, $|\eta| < 1$, inelastic.

$x = 750$, unit elasticity.

62. $p = \sqrt{960 - x}$, $0 \leq x \leq 960$

$$\frac{dp}{dx} = \frac{-1}{2\sqrt{960 - x}}$$

$$\eta = \frac{p/x}{dp/dx} = \frac{\sqrt{960 - x}/x}{-1/2\sqrt{960 - x}} = \frac{2(x - 960)}{x}$$

$$|\eta| = 1 \Rightarrow x = |2x - 1920|$$

$$\Rightarrow x = \pm(2x - 1920)$$

$$\Rightarrow x = 2x - 1920 \quad \text{or} \quad x = -2x + 1920$$

$$\Rightarrow x = 1920 \quad \text{or} \quad x = 640$$

Since $0 \leq x \leq 960$, select $x = 640$.

Elastic: $(0, 640)$

Inelastic: $(640, 960)$

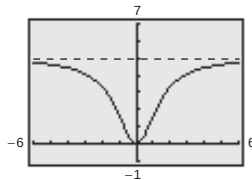
Unit elasticity: $x = 640$

64. $g(x) = \frac{5x^2}{x^2 + 2}$

$$\lim_{x \rightarrow \infty} \frac{5x^2}{x^2 + 2} = 5$$

Horizontal asymptote: $y = 5$

No vertical asymptotes



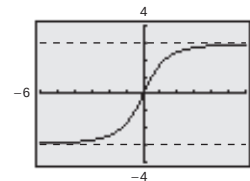
66. $f(x) = \frac{3x}{\sqrt{x^2 + 2}}$

$$\lim_{x \rightarrow \infty} \frac{3x}{\sqrt{x^2 + 2}} = 3$$

$$\lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{x^2 + 2}} = -3$$

Horizontal asymptotes: $y = \pm 3$

No vertical asymptotes

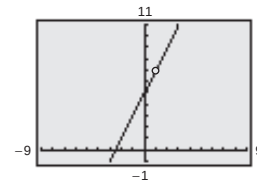


68. $h(x) = \frac{2x^2 + 3x - 5}{x - 1} = \frac{(x - 1)(2x + 5)}{x - 1} = 2x + 5, \quad x \neq 1$

There are no vertical asymptotes. There is a hole in the graph at $(1, 7)$.

There are no horizontal asymptotes since

$$\lim_{x \rightarrow \infty} h(x) = \infty \quad \text{and} \quad \lim_{x \rightarrow -\infty} h(x) = -\infty.$$



70. $\lim_{x \rightarrow -0^-} \left(3 + \frac{1}{x} \right) = 3 - \infty = -\infty$

72. $\lim_{x \rightarrow 3^-} \frac{3x^2 + 1}{(x - 3)(x + 3)} = -\infty$

74. $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 3}{x + 1} = \lim_{x \rightarrow \infty} \frac{3x - 2 + (3/x)}{1 + (1/x)} = \infty$

76. $\lim_{x \rightarrow -\infty} \left(\frac{x}{x - 2} + \frac{2x}{x + 2} \right) = 1 + 2 = 3$

78. $C = 10,000 + 48.9x$, $R = 68.5x$

(a) $\bar{C} = \frac{C}{x} = \frac{10,000}{x} + 48.9$

(b) $\lim_{x \rightarrow \infty} \bar{C} = \48.9

(c) $\bar{P} = \frac{P}{x} = \frac{R - C}{x} = 19.6 - \frac{10,000}{x}$

$\bar{P}(1,000,000) = \$19.59$

$\bar{P}(2,000,000) = \$19.595$

$\bar{P}(10,000,000) = \$19.599$

(d) $\lim_{x \rightarrow \infty} \bar{P} = 19.6$

82. $f(x) = x^2\sqrt{9 - x^2}$

$f'(x) = \frac{3x(6 - x^2)}{\sqrt{9 - x^2}}$

$f''(x) = \frac{6x^4 - 81x^2 + 162}{(9 - x^2)^{3/2}}$

Domain: $-3 \leq x \leq 3$

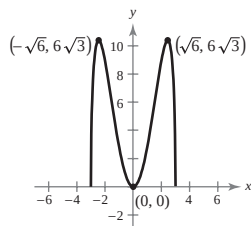
Critical numbers: $x = 0, \pm\sqrt{6}$

Relative maxima: $(\pm\sqrt{6}, 6\sqrt{3})$

Relative minimum: $(0, 0)$

Inflection points: $(\pm 1.5626, 6.2527)$

No asymptotes.



80. $f(x) = 4x^3 - x^4$

$f'(x) = 4x^2(3 - x)$

$f''(x) = 12x(2 - x)$

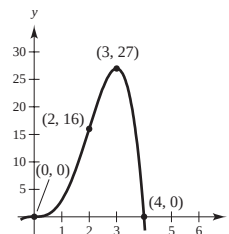
Domain: all real numbers

Range: all real numbers

Intercepts: $(0, 0), (4, 0)$

Relative maximum: $(3, 27)$

Points of inflection: $(0, 0), (2, 16)$



84. $f(x) = \frac{2x}{1 + x^2}$

$f'(x) = \frac{(1 + x^2)2 - 2x(2x)}{(1 + x^2)^2} = \frac{2 - 2x^2}{(1 + x^2)^2}$

$f''(x) = \frac{(1 + x^2)^2(-4x) - (2 - 2x^2)2(1 + x^2)2x}{(1 + x^2)^4}$

$= \frac{-4x(1 + x^2) - 4x(2 - 2x^2)}{(1 + x^2)^3}$

$= \frac{-12x + 4x^3}{(1 + x^2)^3}$

$= \frac{4x(x^2 - 3)}{(1 + x^2)^3}$

Domain: all real numbers

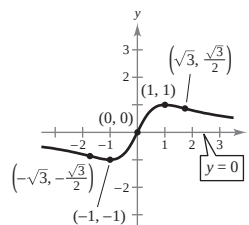
Intercept: $(0, 0)$

Asymptote: $y = 0$

Relative maxima: $(1, 1), (-1, -1)$

Points of inflection: $(0, 0),$

$\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right), \left(-\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$



86. $f(x) = x^{4/5}$

$$f'(x) = \frac{4}{5}x^{-1/5} = \frac{4}{5\sqrt[5]{x}}$$

$$f''(x) = -\frac{4}{25}x^{-6/5} = -\frac{4}{25x^{6/5}}$$

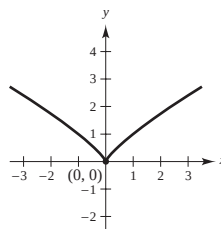
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Intercept: $(0, 0)$

Relative minimum: $(0, 0)$

Concave downward on $(-\infty, 0)$ and $(0, \infty)$



88. $y = (3x^2 - 2)^3$

$$dy = 3(3x^2 - 2)^2(6x) dx$$

$$= 18x(3x^2 - 2)^2 dx$$

90. $y = \frac{2-x}{x+5}$

$$dy = \frac{(x+5)(-1) - (2-x)(1)}{(x+5)^2} dx = \frac{-7}{(x+5)^2} dx$$

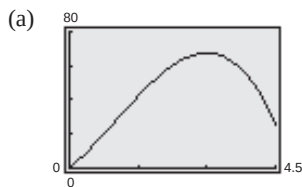
92. $C = 1.5\sqrt[3]{x} + 500, \quad x = 125$

$$dC = 0.5x^{-2/3} dx = \$0.02$$

94. $P = 0.003x^2 + 0.019x - 1200, \quad x = 750$

$$dP = (0.006x + 0.019) dx = \$4.52$$

96. $E = 22.5t + 7.5t^2 - 2.5t^3, \quad 0 \leq t \leq 4.5$



(b) $E'(t) = 0$ for $t = 3$

$$E(3) = 67.5$$

98. $p = 85 - 0.125x$

$$\Delta p = [85 - 0.125(8)] - [85 - 0.125(7)] = -\$0.13$$

$$dp = -0.125 dx = -\$0.13 \quad (\text{since } dx = \Delta x = 8 - 7 = 1)$$