MA 271 PRACTICE PROBLEMS

1. If the line ℓ is parallel to the vector $\vec{v} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, and contains the point $(2, 1, -3)$, then its vector equation is

   A. $\vec{r} = (1 + 2t)\vec{i} - 3t\vec{j} + (-2 + 7t)\vec{k}$ B. $\vec{r} = (2 + t)\vec{i} - 3\vec{j} + (7 - 2t)\vec{k}$
   C. $\vec{r} = (2 + 2t)\vec{i} + (1 - 3t)\vec{j} + (-3 + 7t)\vec{k}$ D. $\vec{r} = (2 + 2t)\vec{i} + (-3 + t)\vec{j} + (7 - 3t)\vec{k}$
   E. $\vec{r} = (2 + t)\vec{i} + \vec{j} + (7 - 3t)\vec{k}$

2. Find parametric equations of the line containing the points $(1, -1, 0)$ and $(-2, 3, 5)$.

   A. $x = 1 - 3t, y = -1 + 4t, z = 5t$ B. $x = t, y = -t, z = 0$
   C. $x = 1 - 2t, y = -1 + 3t, z = 5t$ D. $x = -2t, y = 3t, z = 5t$
   E. $x = -1 + t, y = 2 - t, z = 5$

3. Find an equation of the plane that contains the point $(1, -1, -1)$ and has normal vector $\frac{1}{2}\,\vec{i} + 2\vec{j} + 3\vec{k}$.

   A. $x - y - z + \frac{9}{2} = 0$ B. $x + 4y + 6z + 9 = 0$ C. $\frac{x-1}{\frac{1}{2}} = \frac{y+1}{2} = \frac{z+1}{3}$
   D. $x - y - z = 0$ E. $\frac{1}{2}\,x + 2y + 3z = 1$

4. Find an equation of the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$.

   A. $6x - 11y + z = 5$ B. $6x + 11y + z = 5$ C. $11x - 6y + z = 0$
   D. $\vec{r} = 18\vec{i} - 33\vec{j} + 3\vec{k}$ E. $x - 6y - 11z = 12$

5. Find parametric equations of the line tangent to the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(2, 4, 8)$

   A. $x = 2 + t, y = 4 + 4t, z = 8 + 12t$ B. $x = 1 + 2t, y = 4 + 4t, z = 12 + 8t$
   C. $x = 2t, y = 4t, z = 8t$ D. $x = t, y = 4t, z = 12t$ E. $x = 2 + t, y = 4 + 2t, z = 8 + 3t$

6. The position function of an object is
   $\vec{r}(t) = \cos t\vec{i} + 3\sin t\vec{j} - t^2\vec{k}$
   Find the velocity, acceleration, and speed of the object when $t = \pi$.

	Velocity	Acceleration	Speed
A.	$-\vec{i} - \pi^2\vec{k}$	$-3\vec{j} - 2\pi\vec{k}$	$\sqrt{1 + \pi^4}$
B.	$\vec{i} - 3\vec{j} + 2\pi\vec{k}$	$-\vec{i} - 2\vec{k}$	$\sqrt{10 + 4\pi^2}$
C.	$3\vec{j} - 2\pi\vec{k}$	$-\vec{i} - 2\vec{k}$	$\sqrt{9 + 4\pi^2}$
D.	$-3\vec{j} - 2\pi\vec{k}$	$\vec{i} - 2\vec{k}$	$\sqrt{9 + 4\pi^2}$
E.	$\vec{i} - 2\vec{k}$	$-3\vec{j} - 2\pi\vec{k}$	$\sqrt{5}$

7. A smooth parametrization of the semicircle which passes through the points $(1,0,5)$, $(0,1,5)$ and $(-1,0,5)$ is

A. $\vec{r}(t)=\sin t\vec{i}+\cos t\vec{j}+5\vec{k}, 0\le t\le\pi$
B. $\vec{r}(t)=\cos t\vec{i}+\sin t\vec{j}+5\vec{k}, 0\le t\le\pi$
C. $\vec{r}(t)=\cos t\vec{i}+\sin t\vec{j}+5\vec{k}, \frac{\pi}{2}\le t\le\frac{3\pi}{2}$
D. $\vec{r}(t)=\cos t\vec{i}+\sin t\vec{j}+5\vec{k}, 0\le t\le\frac{\pi}{2}$
E. $\vec{r}(t)=\sin t+\cos t\vec{j}+5\vec{k}, \frac{\pi}{2}\le t\le\frac{3\pi}{2}$

8. The length of the curve $\vec{r}(t)=\frac{2}{3}(1+t)^{\frac{3}{2}}\vec{i}+\frac{2}{3}(1-t)^{\frac{3}{2}}\vec{j}+t\vec{k}$, $-1\le t\le 1$ is

A. $\sqrt{3}$ B. $\sqrt{2}$ C. $\frac{1}{2}\sqrt{3}$ D. $2\sqrt{3}$ E. $\sqrt{2}$

9. The level curves of the function $f(x,y)=\sqrt{1-x^2-2y^2}$ are

A. circles B. lines C. parabolas D. hyperbolas E. ellipses

10. The level surface of the function $f(x,y,z)=z-x^2-y^2$ that passes through the point $(1,2,-3)$ intersects the (x,z)-plane $(y=0)$ along the curve

A. $z=x^2+8$ B. $z=x^2-8$ C. $z=x^2+5$ D. $z=-x^2-8$
E. does not intersect the (x,z)-plane

11. Match the graphs of the equations with their names:

(1) $x^2+y^2+z^2=4$	(a) paraboloid
(2) $x^2+z^2=4$	(b) sphere
(3) $x^2+y^2=z^2$	(c) cylinder
(4) $x^2+y^2=z$	(d) double cone
(5) $x^2+2y^2+3z^2=1$	(e) ellipsoid

A. 1b, 2c, 3d, 4a, 5e B. 1b, 2c, 3a, 4d, 5e C. 1e, 2c, 3d, 4a, 5b
D. 1b, 2d, 3a, 4c, 5e E. 1d, 2a, 3b, 4e, 5c

12. Suppose that $w=u^2/v$ where $u=g_1(t)$ and $v=g_2(t)$ are differentiable functions of t. If $g_1(1)=3$, $g_2(1)=2$, $g_1'(1)=5$ and $g_2'(1)=-4$, find $\frac{dw}{dt}$ when $t=1$.

A. 6 B. 33/2 C. -24 D. 33 E. 24

13. If $w=e^{uv}$ and $u=r+s$, $v=rs$, find $\frac{\partial w}{\partial r}$.

A. $e^{(r+s)rs}(2rs+r^2)$ B. $e^{(r+s)rs}(2rs+s^2)$ C. $e^{(r+s)rs}(2rs+r^2)$
D. $e^{(r+s)rs}(1+s)$ E. $e^{(r+s)rs}(r+s^2)$.

14. If $f(x,y)=\cos(xy)$, $\frac{\partial^2 f}{\partial x \partial y} =$

A. $-xy\cos(xy)$ B. $-xy\cos(xy)-\sin(xy)$ C. $-\sin(xy)$
D. $xy\cos(xy)+\sin(xy)$ E. $-\cos(xy)$

15. Assuming that the equation $xy^2+3z=\cos(z^2)$ defines z implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$.

A. $\frac{y^2}{3-\sin(z^2)}$ B. $\frac{-y^2}{3+\sin(z^2)}$ C. $\frac{y^2}{3+2z\sin(z^2)}$ D. $\frac{-y^2}{3+2z\sin(z^2)}$ E. $\frac{-y^2}{3-2z\sin(z^2)}$

16. If $f(x,y)=xy^2$, then $\nabla f(2,3)=$

A. $12\vec{i}+9\vec{j}$ B. $18\vec{i}+18\vec{j}$ C. $9\vec{i}+12\vec{j}$ D. 21 E. $\sqrt{2}$.

17. Find the directional derivative of $f(x,y)=5-4x^2-3y$ at (x,y) towards the origin

A. $-8x-3$ B. $\frac{-8x^2-3y}{\sqrt{x^2+y^2}}$ C. $\frac{-8x-3}{\sqrt{64x^2+9}}$ D. $8x^2+3y$ E. $\frac{8x^2+3y}{\sqrt{x^2+y^2}}$.

18. For the function $f(x,y)=x^2y$, find a unit vector $\vec{u}$ for which the directional derivative $D_{\vec{u}}f(2,3)$ is zero.

A. $\vec{i}+3\vec{j}$ B. $\frac{i+3\vec{j}}{\sqrt{10}}$ C. $\vec{i}-3\vec{j}$ D. $\frac{i-3\vec{j}}{\sqrt{10}}$ E. $\frac{3\vec{i}-\vec{j}}{\sqrt{10}}$.

19. Find a vector pointing in the direction in which $f(x,y,z)=3xy-9xz^2+y$ increases most rapidly at the point $(1,1,0)$.

A. $3\vec{i}+4\vec{j}$ B. $\vec{i}+\vec{j}$ C. $4\vec{i}-3\vec{j}$ D. $2\vec{i}+\vec{k}$ E. $-\vec{i}+\vec{j}$.

20. Find a vector that is normal to the graph of the equation $2\cos(\pi xy)=1$ at the point $(\frac{1}{6},2)$.

A. $6\vec{i}+\vec{j}$ B. $-\sqrt{3}\,\vec{i}-\vec{j}$ C. $12\vec{i}+\vec{j}$ D. $\vec{j}$ E. $12\vec{i}-\vec{j}$.

21. Find an equation of the tangent plane to the surface $x^2+2y^2+3z^2=6$ at the point $(1,1,-1)$.

A. $-x+2y+3z=2$ B. $2x+4y-6z=6$ C. $x-2y+3z=-4$
D. $2x+4y-6z=0$ E. $x+2y-3z=6$.

22. Find an equation of the plane tangent to the graph of $f(x,y)=\pi+\sin(\pi x^2+2y)$ when $(x,y)=(2,\pi)$.

A. $4\pi x+2y-z=9\pi$ B. $4x+2\pi y-z=10\pi$ C. $4\pi x+2\pi y+z=10\pi$
D. $4x+2\pi y-z=9\pi$ E. $4\pi x+2y+z=9\pi$.

23. The differential df of the function $f(x,y,z) = xe^{y^2-z^2}$ is

A. $df = xe^{y^2-z^2}dx + xe^{y^2-z^2}dy + xe^{y^2-z^2}dz$
B. $df = xe^{y^2-z^2}dx\,dy\,dz$
C. $df = e^{y^2-z^2}dx - 2xye^{y^2-z^2}dy + 2xze^{y^2-z^2}dz$
D. $df = e^{y^2-z^2}dx + 2xye^{y^2-z^2}dy - 2xze^{y^2-z^2}dz$
E. $df = e^{y^2-z^2}(1 + 2xy - 2xz)$

24. The function $f(x,y) = 2x^3 - 6xy - 3y^2$ has

A. a relative minimum and a saddle point
B. a relative maximum and a saddle point
C. a relative minimum and a relative maximum
D. two saddle points
E. two relative minima.

25. Consider the problem of finding the minimum value of the function $f(x,y) = 4x^2 + y^2$ on the curve $xy = 1$. In using the method of Lagrange multipliers, the value of λ (even though it is not needed) will be

A. 2
B. -2
C. $\sqrt{2}$
D. $\frac{1}{\sqrt{2}}$
E. 4.

26. Evaluate the iterated integral $\int_1^3 \int_0^x \frac{1}{x}\,dydx$.

A. $-\frac{8}{9}$
B. 2
C. $\ln 3$
D. 0
E. $\ln 2$.

27. Consider the double integral, $\iint_R f(x,y)dA$, where R is the portion of the disk $x^2+y^2 \leq 1$, in the upper half-plane, $y \geq 0$. Express the integral as an iterated integral.

A. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$
B. $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$
C. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$
D. $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$
E. $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$.

28. Find a and b for the correct interchange of order of integration:
$$\int_0^2 \int_{x^2}^{2x} f(x,y)dydx = \int_0^4 \int_a^b f(x,y)dxdy.$$

A. $a = y^2, b = 2y$
B. $a = \frac{y}{2}, b = \sqrt{y}$
C. $a = \frac{y}{2}, b = y$
D. $a = \sqrt{y}, b = \frac{y}{2}$
E. cannot be done without explicit knowledge of $f(x,y)$.

29. Evaluate the double integral $\iint_R y dA$, where R is the region of the (x,y)-plane inside the triangle with vertices $(0,0)$, $(2,0)$ and $(2,1)$.

A. 2
B. $\frac{8}{3}$
C. $\frac{2}{3}$
D. 1
E. $\frac{1}{3}$.

30. The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1 - x^2$, below by the xy plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral

A. $\int_0^1 \int_0^x (1-x^2)dydx$
B. $\int_0^1 \int_0^{1-x^2} x\,dydx$
C. $\int_{-1}^1 \int_{-x}^x (1-x^2)dydx$
D. $\int_0^1 \int_x^0 (1-x^2)dydx$
E. $\int_0^1 \int_x^{1-x^2} dydx$.

31. The area of one leaf of the three-leaved rose bounded by the graph of $r = 5\sin 3\theta$ is

A. $\frac{5\pi}{6}$ B. $\frac{25\pi}{12}$ C. $\frac{25\pi}{6}$ D. $\frac{5\pi}{3}$ E. $\frac{25\pi}{3}$.

32. Find the area of the portion of the plane $x + 3y + 2z = 6$ that lies in the first octant.

A. $3\sqrt{11}$ B. $6\sqrt{7}$ C. $6\sqrt{14}$ D. $3\sqrt{14}$ E. $6\sqrt{11}$.

33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is

A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.

34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the cone $z = \sqrt{x^2 + y^2}$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.

A. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ B. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

C. $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ D. $\int_{0}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

E. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy\,dz\,dy\,dx$.

35. Do problem 34 in spherical coordinates.

A. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho^3\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$ B. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$

C. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho^3\sin^2\varphi\,d\rho\,d\varphi\,d\theta$ D. $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{32}} \rho^3\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$

E. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho\cos\varphi\,d\rho\,d\varphi\,d\theta$.

36. The double integral $\int_0^1\int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3 dydx$ when converted to polar coordinates becomes

A. $\int_0^{\pi}\int_0^1 r^9\sin^2\theta\,dr\,d\theta$ B. $\int_0^{\frac{\pi}{2}}\int_0^1 r^8\sin^2\theta\,dr\,d\theta$ C. $\int_0^{\pi}\int_0^1 r^8\sin\theta\,dr\,d\theta$

D. $\int_0^{\frac{\pi}{2}}\int_0^1 r^8\sin\theta\,dr\,d\theta$ E. $\int_0^{\frac{\pi}{2}}\int_0^1 r^9\sin^2\theta\,dr\,d\theta$.

37. Which of the triple integrals converts

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$$

from rectangular to cylindrical coordinates?

A. $\int_0^{\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ B. $\int_0^{2\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ C. $\int_0^{2\pi}\int_{-2}^2\int_r^2 r\,dz\,dr\,d\theta$

D. $\int_0^{\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ E. $\int_0^{\frac{2\pi}{2}}\int_{-2}^2\int_r^2 r\,dz\,dr\,d\theta$.

38. If D is the solid region above the xy-plane that is between $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$, then $\iiint_D \sqrt{x^2 + y^2 + z^2}\ dV =$

A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π.

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function f.

1. $\vec{F}(x,y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$.
2. $\vec{F}(x,y) = \frac{x}{y}\,\vec{i} + \frac{y}{x}\,\vec{j}$.
3. $\vec{F}(x,y,z) = ye^z\,\vec{i} + (xe^z + e^y)\vec{j} + (xy+1)e^z\,\vec{k}$.

A. 1 and 2 B. 1 and 3 C. 2 and 3 D. 1 only E. all three

40. Let $\vec{F}$ be any vector field whose components have continuous partial derivatives up to second order, let f be any real valued function with continuous partial derivatives up to second order, and let $\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$. Find the incorrect statement.

A. $\text{curl}(\text{grad } f) = \vec{0}$ B. $\text{div}(\text{curl } \vec{F}) = 0$ C. $\text{grad}(\text{div } \vec{F}) = 0$

D. $\text{curl } \vec{F} = \nabla \times \vec{F}$ E. $\text{div } \vec{F} = \nabla \cdot \vec{F}$

41. A wire lies on the xy-plane along the curve $y = x^2$, $0 \le x \le 2$. The mass density (per unit length) at any point (x,y) of the wire is equal to x. The mass of the wire is

A. $(17\sqrt{17} - 1)/12$ B. $(17\sqrt{17} - 1)/8$ C. $17\sqrt{17} - 1$

D. $(\sqrt{17} - 1)/3$ E. $(\sqrt{17} - 1)/12$

42. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = y\vec{i} + x^2\vec{j}$ and C is composed of the line segments from $(0,0)$ to $(1,0)$ and from $(1,0)$ to $(1,2)$.

A. 0 B. $\frac{2}{3}$ C. $\frac{5}{6}$ D. 2 E. 3

43. Evaluate the line integral

$$\int_C x\,dx + y\,dy + xy\,dz$$

where C is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \le t \le 0$.

A. 1 B. -1 C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. 0

44. Are the following statements true or false?

1. The line integral $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy$ is independent of path in the xy-plane.
2. $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy = 0$ for every closed oriented curve C in the xy-plane.
3. There is a function $f(x,y)$ defined in the xy-plane, such that
$\text{grad } f(x,y) = (x^3 + 2xy)\vec{i} + (x^2 - y^2)\vec{j}$.

A. all three are false B. 1 and 2 are false, 3 is true C. 1 and 2 are true, 3 is false
D. 1 is true, 2 and 3 are false E. all three are true

45. Evaluate $\int_C y^2 dx + 6xy\,dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.

A. 0 B. 4 C. 8 D. 16 E. 32

46. If C goes along the x-axis from $(0,0)$ to $(1,0)$, then along $y = \sqrt{1-x^2}$ to $(0,1)$, and then back to $(0,0)$ along the y-axis, then $\int_C \, xy\,dy =$

A. $-\int_0^1 \int_0^{\sqrt{1-x^2}} y\,dy\,dx$ B. $\int_0^1 \int_0^{\sqrt{1-x^2}} y\,dy\,dx$ C. $-\int_0^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx$
D. $\int_0^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx$ E. 0

47. Evaluate $\int_C \, \vec{F} \cdot d\vec{r}$, if $\vec{F}(x,y) = (xy^2 - 1)\vec{i} + (x^2y - x)\vec{j}$ and C is the circle of radius 1 centered at $(1,2)$ and oriented counterclockwise.

A. 2 B. π C. 0 D. $-\pi$ E. -2

48. Green's theorem yields the following formula for the area of a simple region R in terms of a line integral over the boundary C of R, oriented counterclockwise. Area of $R = \iint_R dA =$

A. $-\int_C y\,dx$ B. $\int_C y\,dx$ C. $\int_C x\,dx$ D. $\frac{1}{2}\int_C y\,dx - x\,dy$ E. $-\int \, x\,dy$

49. Evaluate the surface integral $\iint_S x\,d\sigma$ where S is the part of the plane $2x + y + z = 4$ in the first octant.

A. $8\sqrt{6}$ B. $\frac{8}{3}\sqrt{6}$ C. $\frac{8}{3}\sqrt{14}$ D. $\frac{\sqrt{14}}{3}$ E. $\frac{\sqrt{10}}{3}$

50. If S is the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$, $\vec{n}$ is the unit normal vector on S directed upward, and $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$, then $\iint_S \vec{F} \cdot \vec{n}\,d\sigma =$

A. 0 B. 8π C. 4π D. -4π E. -8π

51. If $\vec{F}(x,y,z) = \cos z\vec{i} + \sin z\vec{j} + xy\vec{k}$, S is the complete boundary of the rectangular solid region bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = \frac{\pi}{2}$, and $\vec{n}$ is the outward unit normal on S, then $\iint_S \vec{F} \cdot \vec{n}\,d\sigma =$

A. 0 B. $\frac{1}{2}$ C. 1 D. $\frac{\pi}{2}$ E. 2

52. If $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$, S is the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{n}$ is the outward unit normal on S, then $\iint_S \vec{F} \cdot \vec{n}\,d\sigma =$

A. -4π B. $\frac{2\pi}{3}$ C. 0 D. $\frac{4\pi}{3}$ E. 4π

53. Evaluate the limit $\lim_{n\to\infty}\left[1+\frac{(-1)^n}{n}\right]$.
A. 0 B. 1 C. -1 D. 2 E. limit does not exist

54. Evaluate the limit $\lim_{n\to\infty}(\sqrt[n]{n}+\frac{1}{n!})$.
A. 0 B. 1 C. e D. $1/e$ E. limit does not exist

55. If $s=\sum_{n=0}^{\infty}\frac{2^{n+1}}{3^n}$, then $s=$
A. 3 B. 6 C. 9 D. 2 E. 4/3

56. If $L=\sum_{n=1}^{\infty}\frac{1}{2^n}+\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$, then $L=$
A. 1/3 B. 2/3 C. 1 D. 4/3 E. 5/3

57. $\sum_{n=1}^{\infty}\frac{1}{(n^2+1)^p}$ converges when
A. $p>1$ B. $p\le 1$ C. $p\ge 1$ D. $p>\frac{1}{2}$ E. $p\le\frac{1}{2}$

58. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^p$ converges for
A. $p\le 1$ B. $p>1$ C. $p<0$ D. $p>0$ E. no values of p

59. Which of the following series converge conditionally?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\qquad\text{(ii)}\ \sum_{n=2}^{\infty}\frac{(-1)^n n}{\ln n}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{(-1)^n n}{e^n}$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

60. Which of the following series converge?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{\frac{1}{4}}}\qquad\text{(ii)}\ \sum_{n=1}^{\infty}\frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{4}{3}\left(\frac{1}{2}\right)^n$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

61. The interval of convergence for the power series $\sum_{n=2}^{\infty}\frac{3^n x^n}{n\ln n}$ is
A. $-\frac{1}{3}\le x<\frac{1}{3}$ B. $-\frac{1}{3}<x\le\frac{1}{3}$ C. $0\le x\le\frac{1}{3}$ D. $-1\le x\le 2$ E. $-1<x<1$

62. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty}\frac{nx^n}{2^n}$$

A. $-\frac{1}{2}<x<\frac{1}{2}$ B. $-2<x<2$ C. $-2\le x\le 2$ D. $-2<x\le 2$ E. $-\infty<x<\infty$

63. The fourth term of the Maclaurin series for $\frac{x^2+3}{x-1}$ is
A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

64. The first three nonzero terms of the Maclaurin series for $f(x)=(1-x^2)\sin x$ are
A. $x-\frac{5}{6}x^3+\frac{31}{150}x^5$ B. $1-\frac{3}{2}x^2+\frac{13}{24}x^4$ C. $x-\frac{7}{6}x^3+\frac{31}{150}x^5$ D . $x^2-\frac{7}{6}x^3+\frac{1}{25}x^5$ E. $x-\frac{7}{6}x^3+\frac{21}{120}x^5$

65. The fourth term of the Taylor series for $f(x) = \ln x$ centered at $a = 2$ is
A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

ANSWERS

1–C, 2–A, 3–B, 4–B, 5–A, 6–D, 7–B, 8–D, 9–E, 10–B, 11–A, 12–E, 13–B, 14–B,

15–D, 16–C, 17–E 18–D, 19–A, 20–C, 21–E, 22–A, 23–D, 24–B, 25–E, 26–B, 27–C,

28–B, 29–E, 30–A, 31–B, 32–D, 33–B, 34–B, 35–A, 36–E, 37–B, 38–C, 39–B, 40–C,

41–A, 42–D, 43–D, 44–E, 45–D, 46–B, 47–D, 48–A, 49–B, 50–E, 51–A, 52–E, 53–B

54– B, 55–B, 56–E, 57–D, 58–E, 59–E, 60–D, 61–A, 62–B, 63–D, 64–E, 65–C