MA 26200 FINAL EXAM (GREEN) SPRING 2015

Name ______________________________ **PUID#** ____________________

Section# ____________ **Class Time** ____________ **Lecturer** ______________________

Exam Rules

1. You may not open the exam until instructed to do so.
2. You must obey the orders and requests by all proctors, TAs, and lecturers.
3. You may not leave during the first 20 min or during the last 10 min of the exam.
4. No books, notes, calculators, or any electronic devices are allowed on the exam, and they should not even be in sight in the exam room. Phones are to be turned off. You may not look at anybody else's test, and may not communicate with anybody else except, if you have a question, with a TA or lecturer.
5. After time is called, you must put down all writing instruments and remain in your seat, while the TAs collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. All violators will be reported to the Office of the Dean of Students.

I have read and understood the exam rules stated above:

STUDENT SIGNATURE: ____

Instructions

1. When told to begin, make sure you have a complete test. There are **14** different test pages, including this cover page. There are 25 problems. Each problem is worth 8 points. The maximum possible score is 200 points. Make sure that you have a **green** answer sheet. Fill in the information requested above. Your PUID# is your student identification number.
2. **Using a #2 pencil**, fill in each of the following items on your **answer sheet**:
   (a) On the top left side, print your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, write in your **4** digit section number (for example 0012 or 0003) and fill in the little circles. **The section numbers are listed below.**
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student I.D. number and fill in the little circles.
   (d) On the bottom right, print your **instructor's name** and the **course number**.
   (e) SIGN your answer sheet. There is no need to fill in the EXAM NUMBER.
3. Do any necessary work for each problem in the space provided or on the back of the pages of this test. No partial credit is given but your work may be considered if your grade is borderline. Circle your answers on this test.
4. **Using a #2 pencil**, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect.
7. Hand in your answer sheet **and** this test to your lecturer or TA.

Here is a list of the section numbers:

0167 - REC 315 MWF 11:30am - **Danielli,** Donatella
0113 - UNIV 101 MWF 09:30am - **Davis,** Rachel
0111 - UNIV 217 TR 03:00pm - **Hedayatzadeh,** M. Hadi
0051 - UNIV 003 TR 04:30pm - **Lai,** Ching-Jui
0133 - UNIV 117 TR 09:00am - **Lipshitz,** Leonard
0031 - REC 315 TR 12:00pm - **Santos,** Juan
0053 - REC 313 MWF 02:30pm - **Wang,** Yuliang
0071 - UNIV 101 MWF 08:30am - **Davis,** Rachel
0163 - UNIV 119 TR 03:00pm - **Donnelly,** Harold
0161 - UNIV 217 TR 04:30pm - **Hedayatzadeh,** M. Hadi
0081 - UNIV 003 TR 03:00pm - **Lai,** Ching-Jui
0165 - ME 1051 MWF 10:30am - **Mohammad,** Asaduzzaman
0131 - REC 315 TR 04:30pm - **Santos,** Juan

1. The solution of the following differential equation satisfying the initial condition $y(0) = 2$ is:

$$8x^3 y dx = (x^4 + 1) dy$$

A. $y = (x^4 + 1)^2 + 1$

B. $y = 2(x^4 + 1)$

C. $y = 2\ln(x^4 + 1) + 2$

D. $y = \dfrac{2}{x^4 + 1}$

E. $y = 2(x^4 + 1)^2$

2. If $y(x)$ is a solution of the exact differential equation

$$(y \cos xy - \sin x) dx + (x \cos xy) dy = 0$$

then y satisfies

A. $\cos xy + y \sin xy = c$

B. $\sin xy + \cos x = c$

C. $y \sin xy + \cos y = c$

D. $x \sin xy + \cos xy = c$

E. $\sin x \cos xy + 2 \cos x = c$

3. Let y be the solution of the initial value problem

$$y' - \frac{1}{2x}y = x, \quad 0 < x, \quad y(1) = \frac{1}{3}.$$

Then $y(4) =$

A. 2
B. 4
C. 5
D. 7
E. 10

4. The subspace of $\mathbb{R}^4$ spanned by $\{(1,2,3,4),(4,3,2,1),(2,0,0,2),(2,4,4,2)\}$ has dimension

A. 0
B. 1
C. 3
D. 2
E. 4

5. A tank initially contains 200 liters pure water. A salt solution containing 10 grams of salt per liter runs in at the rate of 1 liter per second and the well-stirred solution flows out at the rate of 1 liter per second. Find how long it will take in seconds until there are 1000 grams of salt in the tank.

A. $\ln 1000 - e$

B. $200 \ln 2$

C. $200(\ln 3 - \ln 2)$

D. $200(e - 1)$

E. $200 - \ln 200$

6. If the set of vectors $\{(2, 1, a), (5, 3, 7), (6, 5, 7)\}$ is linearly dependent then $a =$

A. 0

B. 1

C. 2

D. 3

E. There is no such value of a.

7. For which values of the constant k, do the vectors $(2, 1, 3k, 4)$, $(0, k-1, 4, -8)$, $(0, 0, 2, 1)$, $(0, 0, k, 4)$ form a basis for $\mathbb{R}^4$?

A. $k \neq 1$
B. $k \neq 0, -1, 8$
C. $k \neq 1, 8$
D. $k = 0, 4$
E. $k = 1, 6$

8. Find the value of k such that $(5, 6k, -2, 2)$ is in the span of $\{(0, 2, 2, 1), (-1, 0, 2, 1), (2, 2, 0, 3)\}$.

A. $k = 1$
B. $k = 3$
C. $k = 6$
D. $k = 7$
E. $k = 10$

9. Let the polynomial $p(x) = \det \begin{bmatrix} 3x & 8 & 7 & 1 \\ x^2 & -2x & 5 & 3 \\ 1 & x & 7 & -2 \\ 2 & 1 & 4 & 7 \end{bmatrix}$. Then the coefficient of x^3 in $p(x)$ is

A. 0

B. 45

C. -45

D. 56

E. -35

10. If $A = \begin{bmatrix} 1+i & -1 \\ 1 & i \end{bmatrix}$ then the **sum** of the entries in the second row of A^{-1} is

A. -1

B. i

C. $1+i$

D. 1

E. $-i$

11. The function $y_1 = e^x$ is a solution of

$$(x-1)y'' - 2xy' + (x+1)y = 0.$$

If we seek a second solution $y_2 = e^x v(x)$ by reduction of order, then $v(x) =$

A. $\frac{1}{3}(x-1)^3 e^{-x}$
B. $\frac{1}{3}(x-1)^3$
C. $(x-1)^2 e^{-x}$
D. $\frac{1}{2}(x-1)^2$
E. $\frac{1}{3}(x-1)^{-3}$

12. A is an $m \times n$ matrix and $\mathbf{b}$ is an $m \times 1$ vector. The equation $A\mathbf{x} = \mathbf{b}$ has **no** solution. Consider the following four statements:

(i) $m \leq n$
(ii) $n \leq m$
(iii) the rank of $A = n$
(iv) the rank of $A \geq m$

How **many** of these statements **must** be true?

A. 0
B. 1
C. 2
D. 3
E. 4

13. Determine all values of k so that $\{k - kx^2, 3 + kx, 2 + x + kx^2\}$ is a basis for $\mathcal{P}_2$, the vector space of all polynomials of degree ≤ 2.

A. $k \neq 1$

B. $k \neq 0, -3, 4$

C. $k \neq -3, 0, 1$

D. $k \neq 1, 2, 4$

E. $k \neq 0, 1, 2, -3, 4$

14. $\mathcal{P}_3$ is the vector space of all polynomials of degree ≤ 3. Let $T : \mathcal{P}_3 \to \mathcal{P}_3$ be the linear operator $D^2 + 1$, i.e. $T(p) = p'' + p$.

Then the dimension of the range of T is

A. 0

B. 1

C. 2

D. 3

E. 4

15. The **sum of** the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 2 & 4 \end{bmatrix}$ is

A. 1
B. 3
C. 5
D. 7
E. 9

16. Determine which one of the sets of vectors is a basis for the solution space of the homogeneous system of equations

$$\begin{aligned} x_1 + 5x_2 + 4x_3 + 3x_4 + 2x_5 &= 0 \\ x_1 + 6x_2 + 6x_3 + 6x_4 + 6x_5 &= 0 \\ x_1 + 7x_2 + 8x_3 + 10x_4 + 12x_5 &= 0 \\ x_1 + 6x_2 + 6x_3 + 7x_4 + 8x_5 &= 0 \end{aligned}.$$

A. $\begin{bmatrix} 1 \\ 0 \\ -6 \\ 0 \\ 6 \end{bmatrix}$,

B. $\begin{bmatrix} 6 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 6 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ -1 \\ 7 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ -1 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -1 \\ 5 \\ 1 \end{bmatrix}$

17. A is a real 3×3 matrix with eigenvalues $1, 1, 3$. Consider the following statements:

(i) the equation $A\mathbf{x} = \mathbf{0}$ has a unique solution

(ii) rank $A = 3$

(iii) the column space of A is $\mathbb{R}^3$

(iv) $A - 2I$ is nonsingular

How **many** of these statements **must** be true

A. 0

B. 1

C. 2

D. 3

E. 4

18. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ are 1 and 2. One of the two eigenspaces has dimension one. This eigenspace has a basis consisting of

A. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 5 \\ -6 \\ 3 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 0 \\ 7 \\ -9 \\ 3 \end{bmatrix}$

E. $\begin{bmatrix} 11 \\ -10 \\ 9 \\ -8 \\ 3 \end{bmatrix}$

19. The solution of the differential equation $(D^4 - 16)y = 0$ satisfying $y(0) = 1$ and $y(x) \to 0$ as $x \to \infty$ has $y''(0) =$

A. -2

B. 2

C. -4

D. 4

E. y'' cannot be determined from the given information.

20. Using the variation-of-parameters method, we know that a particular solution to the differential equation

$$y'' + y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2},$$

is $y_p(x) = u_1(x)\cos x + u_2(x)\sin x$. Then $u_2(x) =$

A. $x \cos x$

B. $-\frac{1}{x^2}$

C. $\ln \cos x$

D. $\sec x$

E. $-\cos x$

21. The general solution to the differential equation $\quad y'' + 9y = 7\cos(4x), \quad$ is

A. $c_1 \sin 3x) + c_2 x \sin(3x)$

B. $c_1 \sin(3x) + c_2 x \cos(3x) + x \sin(4x)$

C. $c_1 \sin(3x) + c_2 \cos(3x) - \cos(4x)$

D. $c_1 \sin(3x) + c_2 \cos(3x) - x \cos(4x)$

E. $c_1 \sin(3x) + c_2 \cos(3x) - \sin(4x) - \cos(4x)$

22. Determine the general solution to

$$(D+1)(D-3)^2(D^2-6D+13)y = 0.$$

A. $c_1 e^{-x} + c_2 e^{3x} + c_3 x^2 e^{-3x}(c_4 \cos 2x + c_5 \sin 2x)$

B. $c_1 e^{-x} + c_2 e^{x} + c_3 x e^{x} + e^{2x}(c_4 \cos 2x + c_5 \sin 2x)$

C. $c_1 e^{-x} + c_2 x e^{x} + c_3 x^2 e^{x} + e^{-3x}(c_4 \cos 2x + c_5 \sin 2x)$

D. $c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x} + e^{3x}(c_4 \cos 2x + c_5 \sin 2x)$

E. $c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x} + x^2 e^{3x}(c_4 \cos 2x + c_5 \sin 2x)$

23. The solution $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ to $\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x}(t)$ satisfying $\mathbf{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ has $x_1(1) =$

A. $2e^3 + e^{-1}$
B. $3e^2 + e^3$
C. $e^2 - e^3$
D. $e^4 - 5$
E. $e + e^2$

24. The real matrix A has an eigenvalue $\lambda_1 = 3 - 2i$ with corresponding eigenvector $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2i \end{bmatrix}$. Then the general solution of the system of differential equations

$$\mathbf{x}'(t) = A\, \mathbf{x}(t)$$

is $\mathbf{x}(t) =$

A. $C_1\, e^{3t} \begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t \\ -\cos 2t \end{bmatrix}$

B. $C_1\, e^{3t} \begin{bmatrix} -2\cos 2t \\ \sin 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t \\ -2\cos 2t \end{bmatrix}$

C. $C_1\, e^{3t} \begin{bmatrix} \cos 2t \\ -2\sin 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t \\ 2\cos 2t \end{bmatrix}$

D. $C_1\, e^{3t} \begin{bmatrix} \cos 2t \\ 2\sin 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} -\sin 2t \\ 2\cos 2t \end{bmatrix}$

E. $C_1\, e^{3t} \begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t \\ 2\cos 2t \end{bmatrix}$

25. The system

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$$

has fundamental matrix

$$\Psi(t) = \begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix}.$$

A particular solution is $\mathbf{x}(t)_p = \begin{bmatrix} e^t & e^{-t} \\ e^t & 3e^{-t} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ where $u_1 =$

A. 2

B. $2t$

C. $-e^t$

D. e^{-t}

E. $-\frac{1}{2}e^{-2t}$