

Homework 1

Due January 19th in class or by 3:20 pm in MATH 602.

1. Find all vectors in $\mathbb{R}^3$ perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$.
2. Find an equation for the plane which is perpendicular to the vector $(1, 2, 3)$ and passes through the point $(1, 1, 1)$.
3. Find the volume of the parallelepiped spanned by $(1, 1, 1)$, $(1, -2, 3)$, and $(-3, 2, -1)$.
4. Find the intersection of the planes with equations $x + 2y + 4z = 7$ and $4x + 2y + z = 7$.

5. Sketch the region given in spherical coordinates by the inequalities

$$0 \leq \rho \leq 1, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi/2.$$

Express this region in cylindrical coordinates.

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7. Expand and simplify $\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2$.

8. Expand and simplify $\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2$.