

Homework 8

Due March 23rd in class or by 3:20 pm in MATH 602.

1. Evaluate

$$\iint_D \sin(x^2 + y^2) dx dy,$$

where D is the region given by $1 \leq x^2 + y^2 \leq 2$, $x \geq 0$, and $y \leq 0$.

2. Evaluate

$$\iint_D e^{y-x} dx dy,$$

where D is the region inside the parallelogram with vertices $(1, 1)$, $(2, 3)$, $(5, 4)$, and $(4, 2)$.

Hint: It may be helpful to use a change of variables which converts the parallelogram into a square.

3. Evaluate

$$\iint_D (x^2 + xy + y^2) dx dy,$$

where D is the region inside the ellipse given by $x^2 + xy + y^2 = 3$.

Hint: It may be helpful to use a change of variables of the form $x = au + bv$ and $y = au - bv$ with constants a and b chosen so as to convert the ellipse to a circle.

4. Let $\alpha = (x^2 + y^2)dx + (x^2 - y^2)dy$. Evaluate and simplify as much as possible the following: $d\alpha$, $*\alpha$, $\alpha \wedge *\alpha$.
5. Let $f = f(x, y)$ be twice differentiable. Write out ddf , $d*df$ and $df \wedge *df$ in terms of the partial derivatives of f , and simplify as much as possible.
6. Let D be the region inside the triangle with vertices $(2, 2)$, $(-2, 2)$, and $(0, 10)$. Find the flux of the vector field

$$(e^{\cos(x^2)} + e^{\cos(y^2)}, x^2 + y^2)$$

outward through the boundary of D .