

MA 362 first midterm review problems

- The first midterm will be on Monday, February 6th, from 8:00 to 9:30 pm, in CL50 224.
 - It will cover all the material from the classes and homework up to that date.
 - Most of the problems on the exam will be closely based on ones from the list below (but the actual exam will be much shorter).
 - For each problem, you must explain your reasoning.
 - Note that these are not arranged in order of difficulty!
1. Let $(a, b, 0)$, $(0, a, b)$, and $(a, 0, b)$ be three (not necessarily distinct) points in $\mathbb{R}^3$, where a and b are real numbers.
 - (a) For which real values of a and b are the points collinear?
 - (b) For which values are they coplanar with the origin?
 - (c) How do the answers change if the third point is replaced by $(b, 0, a)$?
 2.
 - (a) Find a normal unit vector to the plane in $\mathbb{R}^3$ given by $x - 2y - 2z = 1$.
 - (b) Find the intersection of this plane with the line perpendicular to it and passing through $(1, 2, 3)$.
 - (c) What is the distance between the plane and the point $(1, 2, 3)$?
 3. What is the distance between the two planes given by the equations $x + 2y + 3z = 0$ and $x + 2y + 3z = 4$?
 4. What is the cosine of the angle between the diagonal of a cube and one of the edges? What about between the diagonal of a cube and the diagonal of one of its sides?
 5. What are the smallest possible numbers a and b such that
$$\|x + y\|\|x - y\| \leq a\|x\|^2 + b\|y\|^2,$$
for any x and y in $\mathbb{R}^n$?
 6.
 - (a) Find the equation of the sphere in $\mathbb{R}^3$ with center $(1, 1, 1)$ and passing through $(2, 2, 2)$.
 - (b) What is the intersection of this sphere with the xy -plane?
 7. Find the equation for the surface consisting of those points in $\mathbb{R}^3$ which

are equidistant from the y -axis and the xz -plane. Sketch and describe the surface.

8. Let n be an integer. For which values of n does the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^{2n} + y^2}$$

exist?

9. Let

$$f(x, y) = \begin{cases} x, & \text{if } |y| > |x|, \\ -x, & \text{if } |y| \leq |x|. \end{cases}$$

- (a) At which points (x_0, y_0) do the partial derivatives of f exist? What are the values of the partial derivatives at those points?
- (b) At which points is f continuous?
- (c) At which points is f differentiable?

10. Let

$$f(x, y) = \begin{cases} \frac{(x^2 - y^4)^2}{(x^2 + y^4)^2}, & \text{if } (x, y) \neq (0, 0), \\ 1, & \text{if } (x, y) = (0, 0). \end{cases}$$

- (a) Show that the directional derivatives of f exist at $(0, 0)$ in all directions. What are their values?
- (b) Is f continuous at $(0, 0)$?
- (c) Is f differentiable at $(0, 0)$?

11. Let $f = f(u, v, w)$ be a differentiable function $\mathbb{R}^3 \rightarrow \mathbb{R}$, and let

$$g(x, y) = f(x - y, x + y, xy)$$

- (a) If $(4, 10, 21)$ is a critical point of f , which point must be a critical point of g ?
- (b) Calculate $\partial_x \partial_y g$ in terms of the derivatives of f .

12. A train track is to be built up a hill given by the graph in $\mathbb{R}^3$ of the function $f(x, y) = 10 - x^2 - 2y^2$. What are the possible directions the

track can head starting at $(1, 0)$ so that it is going uphill at a slope of $1/2$?

13. Suppose that $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ is differentiable and satisfies

$$\nabla f(1, -e, e) = (1, 2, 3),$$

and that $z = z(x, y)$ is given by the implicit equation

$$x - y + z + \log z = 2 + 2e,$$

where x and y are independent variables. If $g(x, y) = f(x, y, z(x, y))$, what is $\nabla g(1, -e)$?

Formula sheet

- Cylindrical coordinates in $\mathbb{R}^3$ are given by $x = r \cos \theta$, $y = r \sin \theta$, $z = z$. Spherical coordinates are given by $x = \rho \sin \varphi \cos \theta$, $y = \rho \sin \varphi \sin \theta$, and $z = \rho \cos \varphi$.